PHYSICAL DETERMINATIONS

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LABORATORY INSTRUCTIONS

FOR THE DETERMINATION OF PHYSICAL QUANTITIES

CONNECTED WITH

GENERAL PHYSICS, HEAT

ELECTRICITY AND MAGNETISM

LIGHT, AND SOUND

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PREFACE

In compiling this little volume my endeavour has been to supply outline directions which might enable a class of students to proceed with work until the demonstrator could give individual instruction to each group.

Discussion of detail is omitted, but I hope a clear statement of the quantity to be determined is generally given. The comparatively large number of determinations dealt with will perhaps be wide enough to permit of teachers mapping out their own courses; for few students will, I fear, find time to do all.

I venture to hope that the collection may prove of use in Technical Schools; and, so far as the operations are concerned, it includes most of those which have been given at the London University Intermediate and Final B.Sc. examinations.

In conclusion, my heartiest thanks are due, and given, to Professor H. Tomlinson, F.R.S., for reading most of the sheets, and for many kind suggestions during their production.

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July, 1901.

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PHYSICAL DETERMINATIONS

PART I

GENERAL MEASUREMENTS

1.

The Determination of Lengths.—The usual method of measuring the distance between two points on the same plane surface is to apply a graduated scale directly, noting the number of scale divisions between the points. It is convenient to adjust the zero of the scale to one point, and to read the number of divisions between it and the dividing line which most nearly coincides with the other point.

This number is the length between the points in terms of the scale divisions, as nearly as it can be directly measured from the scale. Generally, however, it is possible to estimate by what fraction of a scale division the point is beyond or short of the noted dividing line: this fraction being added or deducted as necessary gives the length, by estimation, to a higher degree of accuracy. If the scale zero is at the extreme end of the scale, it is better to adjust, not the zero, but one of the dividing lines near the end, to the first point, and to take the difference of the readings at the two points as the length between them.

It is advantageous to place the scale in such a position that the dividing lines touch the plane surface in which the points lie; by this means parallax, *i.e.* apparent relative motion between the points and the dividing lines when the eye is moved, is avoided.

In cases where it is impossible to lay a scale from point to point, the **beam compass** may be used. This instrument consists of a bar along which slide two pieces bearing pointed legs, and which may be clamped in any position by screws. The pointed legs are adjusted to the two points between which the length is required, and are then themselves applied to a scale.

The length of curved lines may be very approximately obtained by stepping along them with dividers opened a short distance and counting the total number of steps. A straight line is then drawn, and the dividers, still opened the same distance, are stepped along it. The total length of a considerable number of these steps is measured by a scale, and this divided by the number of steps gives the distance the dividers are open. The latter multiplied by the number of steps in the curve is the length of the curve.

The diameters of cylinders, distances between parallel planes, etc., may conveniently be measured by calipers.

Outside calipers consist of two curved arms, hinged together at one end and bearing inwardly directed points at the other. The hinge is stiff, so that they may be moved or laid down without altering the position of the arms. The body to be measured is placed between the points, which are adjusted by tapping the outside of an arm or the point on a bench until they just pass over it. The calipers are then applied to a scale, and the distance between the points noted.

Inside calipers are similar, but have the arms nearly straight and their points turned outwards. They are placed between the points, the distance of which apart is to be found, and adjusted until they just touch without any play, after which the points are applied to a scale.

A more accurate instrument for the same purpose is the **vernier calipers.** This consists of a graduated bar with an arm at right angles at one end. Along the bar slides another arm, which is sometimes furnished with a trigger-like clamp to fix it in any position along the bar. In other forms of the instrument a sliding piece attached to the arm is fixed by a screw when approximately in its correct position, and the fine

adjustment made by means of another screw at the end of the bar.

Outside measurements may be made with the inner faces of the arms, inside measurements with the outer faces. The sliding arm usually carries verniers (see 4), reading on the graduated bar. In use the zero error, *i.e.* the reading when the arms are closed and in contact with each other, is firstly noted. This, if negative, is added to the later readings, and, if positive, is deducted.

Outside diameters may be very approximately found by winding a thread an exact number of times round the circumference in a single layer, measuring the axial length, l, of the winding, and the total length of the turns when unwound. If the latter be L, and the number of turns be N—

$$\text{Diameter} = \frac{L - \frac{l^2}{2L}}{\pi N}$$

or, if $\frac{l}{L}$ be small, approximately—

$$\text{Diameter} = \frac{L}{\pi N}$$

Measurements of small thicknesses and diameters may be made by means of a wedge, or a plate with a wedge-shaped slot cut out, graduated on one side.

The wedge is applicable to the measurement of distances between plane faces, the slot to those of thicknesses and diameters.

The object of which the thickness is required is inserted in the slot until it makes contact with each side. Let L be the length of slot, W its maximum width, and l the length from the end of the slot to the point of contact. Then, by similar triangles—

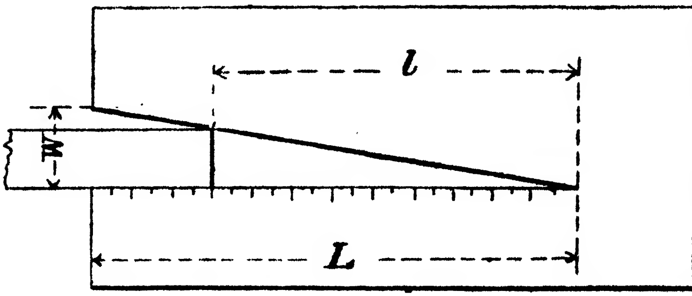
$$\text{Width of object} = \frac{W}{L}l = l \tan \theta$$

where θ is the angle between the sides of the slot.

The diameter of a circular rod placed in the slot is given by—

$$\text{Diameter} = 2 \tan \frac{\theta}{2} \times l$$

The values of $\tan \theta$ and of $2 \tan \frac{\theta}{2}$ may be found once for all as a constant for any particular slot. Similarly, if the wedge



be inserted between two surfaces for a length, l, the distance between the points at which its edges make contact is—

$$\frac{W}{L} l = l \tan \theta$$

It is advantageous that the edges of the wedge should be bevelled. If, as is often the case, the point of it be truncated off, the lengths L and l are measured from that position at which it would exist.

2.

To determine Thicknesses by Means of the Micrometer Gauge.—The micrometer gauge consists of a short metal bar with an arm projecting at right angles from each end. Through one of these is screwed a short plug, and through the other passes a screw coaxial with the plug. The ends of both screw and plug are plane surfaces perpendicular to the axis. The pitch of the screw is half a millimetre, and a scale engraved upon the arm through which it passes permits of its position being read. For this purpose the head of the screw passes over the scale. The latter is usually

divided into half millimetres, so that one complete turn of the head carries it along one division. The circumference of the head itself is divided into fifty parts, the reading of which is taken on the base line of the fixed scale, thus permitting the measurement to be made directly to the nearest hundredth of a millimetre, and by estimation to a thousandth. Owing to the uncertainty of the pressure put on the measured body, however, and sundry other circumstances, the estimation to thousandths is of very little value in many cases.

In making a measurement, the screw is run in until its end is in contact with the plug. Some instruments are fitted with Breguet heads, which slip round when contact occurs, preventing undue force being applied. When this is absent, the screw should be turned very lightly into contact. The zero error, *i.e.* the reading on the head, is then noted. In a perfectly adjusted gauge the reading would be o, but this is usually not the case. If the zero is passed in obtaining contact, the error is positive, and must be added to the length measured later; if the zero is not reached, the error is negative, and is deducted.

The screw is then wound out, the body to be measured put between it and the plug, and the screw wound into contact with the body. The scale and head readings are noted and the zero correction made.

Measurements should be made several times, in determining the diameters of wires, etc., also in different places and directions.

3.

To determine Thicknesses and Curvatures by Means of the Spherometer.—The spherometer consists of three pointed legs, rigidly attached to a frame, and having the same relative position to one another as the points of an equilateral triangle. In the centre is a fourth leg, equidistant from the others, and capable of being screwed up or down by a head. To this head is attached a disc with a bevelled edge, the vertical position of which may be read from a scale on the frame, thus

indicating the vertical distance between the point of the central leg and the plane passing through the points of the other three.

The fixed scale on the frame is usually divided into half millimetres, and the screw is raised this distance, or lowered the same, in exactly one revolution. The circumference of the disc itself is usually divided into fifty parts, each subdivided into tenths.

Consequently, each small division of the disc passing the edge of the fixed scale indicates one thousandth of a millimetre alteration of position to the point.

The fixed scale usually has its zero at the middle of its height, so that distances of the central leg, either above or below the plane of the others, may be read directly. In the former case, the disc divisions are taken right-handedly; in the latter, left-handedly.

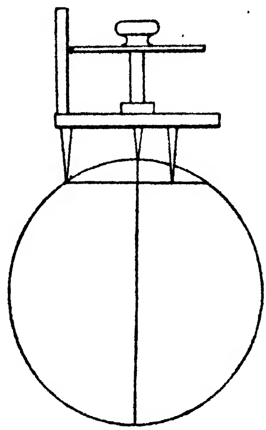
In making a measurement the three legs should firstly be placed on a perfectly plane surface, to supply which a glass plate is provided with the instrument, and the central leg screwed down till it just makes contact. The twisting of the head should proceed very slowly as this position is approached, and the reading noted when it is obtained. The mean of several such readings should be taken as the zero error.

If contact occurs when turning left-handedly, before zero is indicated, the zero error is deducted from upward, and added to downward, measurement readings. The reverse corrections are made if the zero division is passed.

To measure the thickness of a plate, the central leg is screwed up, and the plate placed on the glass beneath it. The leg is then screwed down gently as before, and the scale and disc are read. This reading, with the necessary correction for zero error, is the thickness of the plate.

In determining the curvature of a spherical surface, the three legs are placed upon it, and the central leg screwed into contact. Let the reading, corrected for zero error, be l, and let the distance between the points of two of the fixed legs be L. Then the radius of the circle upon the circumference of which the three legs lie is equal to $\frac{L}{2 \sin 60^\circ} = \frac{L}{\sqrt{3}}$

But (*Euc.* iii. 35) if in a circle a diameter be drawn, also any chord perpendicular to it, then the product of the two lengths into which the diameter is divided by the chord equals half the length of the chord squared.



But the lengths into which the diameter is divided are l and $(2R-l)$, where R is the radius. Hence—

$$l(2R-l)=\left(\frac{L}{\sqrt{3}}\right)^2$$

$$\text{and } R=\frac{L^2}{6l}+\frac{l}{2}$$

To test the sphericity of a surface, the instrument should be placed on various parts, and the curvatures, viz. l divided by $\frac{2L}{\sqrt{3}}$, compared for each.

4.

On the Use of the Vernier.—A vernier is used to determine the position of a point relatively to a fixed scale with an accuracy which is otherwise impossible, except by the use of a scale with inconveniently fine subdivisions.

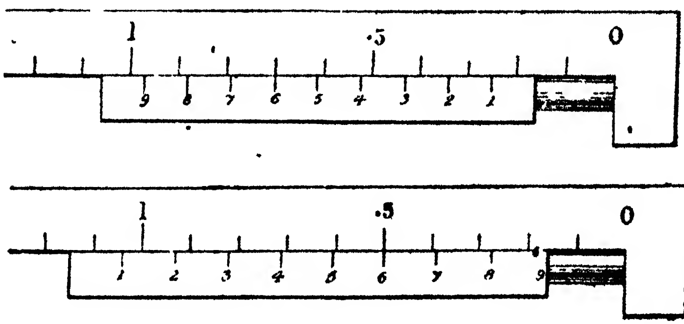
It is a small slider moving along the fixed scale, and is itself graduated in such a way that n of its divisions are equal in length to $n-1$ of those on the scale.

When, therefore, its zero coincides with a scale line, its first dividing line is $\frac{1}{n}$ of a scale division short of the next scale line, its second is $\frac{2}{n}$ short of the scale's second, etc.

Supposing the point of which the position is to be found to lie between two of the scale marks, and the zero of the vernier to be adjusted to coincidence with it.

If now the first vernier line coincides with a scale line, it

follows that the point is $\frac{1}{n}$ of a scale division beyond the lower of the scale lines between which it lies. Similarly, if the *m*th line of the vernier coincides with a scale line, the point with which its zero coincides is $\frac{m}{n}$ of a scale division beyond the next scale line below it.



It may happen that no line of the vernier exactly corresponds in position with one on the scale. In such a case the vernier reading to the scale line most nearly coinciding with one of its own is estimated. Whatever this reading be, integral or fractional, it is the number of *n*ths of a scale division to be added to the scale reading.

In the form of vernier described the vernier and scale numbering run in the same direction. In some forms, however, n of the vernier divisions are equal to $n + 1$ of those on the scale, and their numbering is in the reverse direction. The method of use is the same in both forms.

Verniers on scales of length commonly read to tenths of the smallest scale divisions, those on rings divided for angular measurements usually to minutes.

In the absence of a model, a scale and vernier should be constructed on cardboard, the scale divided into centimetres, the vernier reading to a millimetre. In either case the lengths of several rods should be measured.

5.

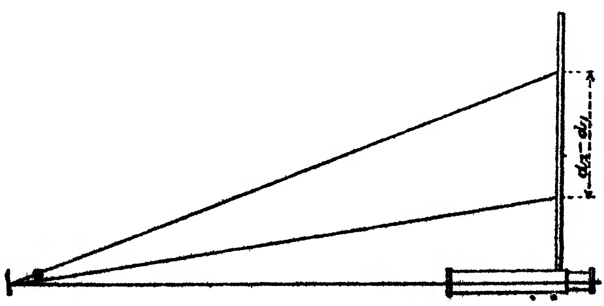
To determine Thicknesses by Means of the Optical Lever.—The optical lever is a bar of a few centimetres length supported on two legs at the centre and one at each end. At the centre is a small mirror turning about a horizontal axis at right angles to the length of the bar.

A telescope furnished with cross-wires is directed towards the mirror, and a scale is erected vertically above it in the region of the cross-wires. Adjustment is then made until the reflection of the scale is seen in the mirror, without parallax, when the lever stands upon a perfectly plane surface.

The object of which the thickness is to be determined is placed under the central pair of legs, one end of the lever is depressed, and the scale division which coincides with the horizontal cross-wire noted. The other end of the lever is then depressed, and the same observation repeated.

Let the measured distance between the mirror and the scale be L, and l that between the two end legs of the lever. Then, if t be the thickness of the object measured, and θ the angle made by the lever with the horizon when tilted—

$$t = \frac{l}{2} \sin \theta$$



Now, the total angle turned through by the mirror is 2θ, and

that by the reflected ray is 4θ, so that if $d_2 - d_1$ be the length of scale between the two noted readings—

$$\tan 4\theta = \frac{d_2 - d_1}{L}$$

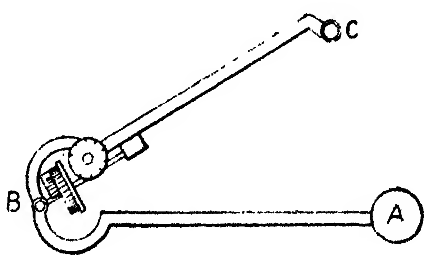
very approximately, since θ is small. Assuming, then, that θ, sin θ, and tan θ are equal—

$$t = \frac{(d_2 - d_1)l}{8L}$$

The distance between the telescope and lever may conveniently be from 1 to 4 metres.

6.

To determine Areas by Means of the Planimeter.—Several types of planimeter exist, one of the most generally used being that of Amsler. This essentially consists of two arms, AB and BC, jointed at B. At A is a pointed leg which acts as a fixed centre when the instrument is used, and at C is a point which is moved along the line circumscribing the



measured area. A rolling wheel attached to BC partly slides and partly rolls over the plane in which the area lies. By means of a worm, motion is given to a disc on BC which records tens of square inches, while the edge of the roller itself is graduated into square inches and tenths of the same, reading by the vernier to hundredths. The circle traced by the point

at C, when the arm BC is perpendicular to AB and the whole rotates about A, is termed the "datum" circle.

In measuring an area, A is fixed, and C adjusted at some noted point on the boundary of the area. The reading of disc and wheel are noted, and the point is carried once in a clockwise direction round the area to the exact spot from which it started. The reading is again noted, the difference between it and the previous one being the required area. In the case of small areas, it is advisable to carry the point round N times, and to divide the recorded area by N.

It is to be noted, however, that if the datum circle lies entirely within the boundary of the area, the area of this circle, which is recorded on the instrument, must be added to the reading. If, on the other hand, the area measured lies wholly within the datum circle, the reading must be subtracted from that circle's area to give the required area.

The areas of several figures, both regular and irregular, should be determined, and the areas of the former, as found by calculation, compared with the planimeter measurement of the same.

It is to be noted that areas passed round in a counter clockwise direction are recorded negatively, the final being less than the initial reading. Thus, if a line crossing itself—as, for example, an 8-shaped figure—be traversed, the difference of initial and final readings is the difference between the areas enclosed by the loops.

Special care should be given in using this instrument not to handle or touch the recording dial and wheel.

7.

To determine Differences of Height by the Cathetometer, and to standardize the Eye-piece Scale.—The cathetometer essentially consists of a telescope, furnished with cross-wires, and vertically movable on a bar, with a scale from which the distances moved may be read.

Some instruments possess elaborate adjustment fittings,

many carry only a vernier and eye-piece scale. The latter form is assumed to be now used.

The telescope may be adjusted to a horizontal position very approximately by laying a level along its tube, and adjusting the stand by levelling-screws if they be present, or by packing under the base if otherwise.

The scale in the eye-piece should then be turned vertical, and adjusted in this position by focussing the telescope on a plumb-line.

A needle should then be placed vertically in a clamp, substituted for the plumb-line, and moved until it is in focus, the image of its point coinciding with one end of the scale. When in this position the eye-piece must be adjusted until there is no parallax, *i.e.* relative motion between the image and scale, when the eye is moved.

The reading of the scale on the stand is now noted.

The telescope is then moved up or down until the image of the point coincides with the other end of the eye-piece scale, and the reading of the stationary scale again observed.

Let the difference between the two readings taken from the fixed scale be N, and the number of eye-piece scale divisions the image of the needle point moved over be n. Then—

$$\text{One eye-piece division} = \frac{N}{n} \text{ fixed-scale divisions}$$

In measuring the vertical distance between two points, the telescope is focussed on one of them, and the readings of both scales noted. If the other point is also in the field of view, the distance between the two is read from the eye-piece scale. Otherwise, the telescope is moved up or down until the other point comes into view, and both scales are again read.

The difference between these readings is the vertical distance between the points.

Proceeding in this way, several vertical distances should be measured, and the divisions on the fixed scale of the instrument compared with those of another scale placed vertically in front of the telescope.

8.

To determine Masses by Means of the Balance.—The balance consists of a beam supported on a central knife-edge, and carrying others at its ends from which pans are suspended. The central knife-edge is so arranged that it may be raised or lowered, thus putting the balance into action, or allowing the beam to rest upon supports. The mechanism effecting this is actuated by a lever in front.

A pointer attached to the middle of the beam, and moving over a scale at the bottom of the supporting column, indicates the position of the beam relatively to the stand.

To facilitate exact weighings, one arm of the beam is graduated, and a wire rider is so arranged that it may be set astride of the former at any distance along the scale by means of a rod projecting outside the instrument's case. When so set, the scale reading indicates the number of milligrams by which the weights in the pan are virtually increased.

The entire balance is enclosed in a case, which protects it from draughts, and is only opened to add or remove the weights; in simpler forms of instrument, a place as free as possible from air currents should be selected for work.

Assuming the general adjustments to be correct, the base is levelled by means of a spirit-level and levelling-screws, the pointer being brought to the central division of the scale, either by adjustment of a metal flag at the middle of the beam, or by means of screwed weights at its end. Unless already existing, a curve of sensitiveness should then be constructed. In doing this, and in all weighings, care must be taken that (*a*) the beam is always lowered before making any alteration to the masses in the pans; (*b*) the raising of the beam is performed slowly, and that, if the pointer swings quickly to one side, it is immediately lowered, and the weights readjusted.

To construct a curve of sensitiveness, the position of the pointer when both pans are empty is noted. A small weight is added to the right-hand pan, and the displacement of the pointer in scale divisions observed. Hence the weight in milligrams producing a displacement of one scale division,

i.e. the sensitiveness, is calculated. Equal small weights are then placed in each pan, and the displacement produced by a small addition to the right-hand one observed. Dividing the small additional weight by the number of scale divisions displacement, the sensitiveness for the particular load in the pan is obtained. Proceeding in this way for various loads up to the greatest the balance is constructed to carry, the sensitiveness for each is determined. A curve should then be plotted with loads as abscissæ and the sensitiveness as ordinates.

The mass of any body is found by placing it in the left-hand pan, and a weight which, as nearly as may be judged, will balance it in the other. In general no one weight will exactly do this, but by trial two may be selected, one of greater, the other of less, mass than the body. The smaller is placed in the pan, and in a similar way other less weights are added, until the pointer oscillates within the limit of the scale when the beam is raised. The weights themselves should not be touched by the fingers, but always moved by means of the forceps provided.

Three observations should now be made of the pointer's position at the end of the swing. Let a and c be the scale divisions reached by the pointer in two successive swings to the left, the intermediate swing to the right being b, and all divisions being counted from the extreme left of the scale toward the right hand.

Then, if the pointer were allowed to come to rest, it would do so at the midpoint of the right- and mean left-hand positions, or at the $\frac{\frac{1}{2}(a+c)+b}{2}$ division. Thus, if when the balance is unloaded the pointer rest at division d, a weight $\left(\frac{\frac{1}{2}(a+c)+b}{2}-d\right)m$ would have to be added to those actually in the pan to produce exact balance, m being the sensitiveness for the particular load on the balance. Due attention must, of course, be paid to the sign of this addition. The sensitiveness is, of course, found by reference to the previously constructed curve.

In an exact weighing, a correction must now be applied for the buoyancy of the air displaced by the body and the weights. Let V be the volume in cubic centimetres of the body, and v

that of the weights. Then the weight as found above must be increased by $(V - v) \times$ density of air per cubic centimetre. In nearly all cases the density of air may be taken as 0·0013 gram, and the volume of the small platinum weights may be neglected. The volumes may be calculated if the dimensions be known, or found by any of the usual methods described elsewhere.

If the balance be out of adjustment, the following conditions may exist, and be corrected for as follows:—

(*a*) The scale-pans may not be of equal weight. The body is firstly placed in one pan and counterbalanced by weights W_1; the pans are then interchanged, and the body, still at the same end of the beam, is found to be counterbalanced by weights W_2. Then the pan in which the body was originally placed is heavier than the other by an amount $\frac{1}{2}(W_1 - W_2)$, and the true weight of the body is $W_1 - \frac{1}{2}(W_1 - W_2)$. Due regard must be paid to the sign of the correction.

(*b*) The beam-arms may not be of equal length. In this case the pans being adjusted to have equal weight, a body is weighed at one end of the beam, and the apparent weight found to be W_1. It is then placed in the other pan, and found to have an apparent weight W_2.

Let W be the true weight of the body, L_1 the length of that arm from which it first hung, L_2 the length of the other. Then—

$$L_1 \times W = L_2 \times W_1, \text{ and } L_2 \times W = L_1 \times W_2$$

Consequently—

$$\frac{L_1}{L_2} = \sqrt{\frac{W_1}{W_2}}, \text{ and } W = \sqrt{W_1 \times W_2}$$

9.

To determine the Density of Air by a Method of Weighing.—A large glass globe, fitted with a cock and filled with air, is suspended from one arm of a balance and weighed, or, preferably, counterbalanced by a similar globe. In the latter case the corrections for the external air displaced are avoided. During this operation the cock of the globe may be open, so that the contained air is at the same pressure as that outside,

and this pressure should be noted from a barometer. The globe is then removed from the balance, and partially exhausted by an air-pump furnished with a manometer. After exhaustion, it should remain connected to the pump for a few minutes, in order that the residual air inside may acquire the external temperature, care being taken not to handle or otherwise unduly warm it. The manometer is then read and the cock closed.

The pressure of the contained air has now the same ratio to the original pressure that the difference between the barometer and manometer pressures has to the barometer pressure.

The globe is again weighed, and the difference between this and the original weight is the weight of air pumped out. Let this be w grams.

The internal volume of the globe must now be found either by means of the volumenometer (Art. **51**), or by filling it with water and measuring or weighing the quantity contained. If the latter course be adopted, the volume is given directly, with but a small error, on account of some adhering to the measuring vessel.

Now, the air in the globe after exhaustion, if compressed till it regained its original pressure, would occupy a volume v such that—

$$v = V\left(\frac{H-h}{H}\right)$$

where V is the internal volume of the globe, H the barometric pressure, and h that of the manometer.

Therefore a volume $V\left(1 - \frac{H-h}{H}\right)$ of air at the temperature t of the atmosphere, and at pressure H, weighs w grams, and from this the density, *i.e.* the weight of one cubic centimetre, at 0° C. and 760 mm. pressure may be found, viz.—

$$\frac{w}{V\left(1 - \frac{H-h}{H}\right)} \times \frac{760}{H} \times \frac{273+t}{273}$$

The determination may be more satisfactorily made by

compressing air into the flask by a bicycle pump, if a three-way connection be made for the globe, pump, and manometer. By this means a greater difference of pressure and of weight may be obtained.

The manometer used for pressures less than that of the atmosphere may be a glass tube dipping into a vessel of mercury, and in this case h is simply the difference of level inside and outside the tube, H being taken as the barometric height. When the pressures are greater than this, a U-tube, with one limb of considerable length, is necessary, the difference of levels in the limbs being noted.

10.

To determine the Mean Internal Diameter of, and to calibrate, a Glass Tube.—The ends of the tube should be ground flat and at right angles to the axis by holding the tube near the lower end and rubbing it upon a smooth flagstone plentifully wetted. The tube should be held vertically during the grinding.

It is then, after cleaning and drying, filled with mercury by immersion in a trough of the liquid. For this purpose a long trough about three-quarters of an inch deep and of equal width, cut in a flat board, is convenient.

The tube after filling is withdrawn by a finger placed over each end, or by holding small glass plates over them, and the mercury is poured out into a previously balanced beaker.

The weight of the mercury is found, and its weight, divided by the density at the existing temperature, gives the internal volume of the tube. But—

$$\text{Volume} = \frac{\pi \times \text{diameter}^2 \times \text{length}}{4}$$

hence the mean diameter $= 2\sqrt{\dfrac{\text{volume}}{\pi \times \text{length}}}$

A slight error is introduced when the tube is closed by the

fingers, owing to their entering a small distance, but unless the tube be very short indeed this is negligible.

To calibrate the tube, *i.e.* to compare the sectional areas at various points of its length, a thread of mercury about one or two centimetres long should be introduced at one end. The tube is then clamped horizontally under a reading microscope, and the length of the thread measured. The distance of its centre of length from the end of the tube should also be noted to the nearest millimetre by a scale. The reading microscope is a compound one furnished with cross-wires, and movable along a horizontal scale, from which the distances moved may be read. The cross-wire should be focussed without parallax upon the end of the thread's meniscus. This is not theoretically the correct position, for the object is to obtain the length of a thread with plane ends. Attempts to estimate this may be made in adjusting the cross-wires, but the advantage gained by most observers is probably counterbalanced by the consequent want of a definite point on which to focus.

The observations mentioned having been made and noted, the thread should be displaced through nearly its own length along the tube. This is conveniently done by attaching a few inches of rubber tube to that end of the glass tube at which observations are started. By compressing this from the open end toward the other, the thread may be driven along. The same observations as before are made, and the operations repeated until the thread reaches the other end of the tube. It is then poured into a previously balanced beaker and weighed.

This weight, divided by the product of the density at the existing temperature and the length of thread, gives the mean sectional area of tube for each position of the bead's centre.

Results should be tabulated thus:

Distance from end of tube to centre of thread.	Length of thread.	Sectional area.

A line should also be drawn on squared paper equal in

length to the tube, and at lengths along it corresponding to the positions of the thread's centre, ordinates erected to a large scale representing the sectional area minus some constant number at each. An even curve should then be drawn through the extremities of the ordinates. One end of the tube and of the curve should be similarly marked to prevent the curve corrections being applied from the wrong end.

The graduations of burettes, measuring vessels, etc., may be calibrated by filling with water or mercury, running out and weighing the volumes between noted graduations, and dividing by the density of the liquid. Mercury is by far the more satisfactory liquid to use, as it does not wet the surfaces; in the case of water, a certain amount always adheres, so that the entire volume between divisions is not run out.

In reading the level of a liquid within a graduated tube any difficulty introduced by the meniscus may be avoided by using an Erdmann's float. This is a short glass tube weighted at one end and bearing a transverse mark. The position of the mark relatively to the graduations is read.

11.

To determine the Volume of a Solid by the Displacement of Liquid.—In this method, which is only applicable to solids which are denser than some convenient liquid, a siphon is required with its short arm drawn out to a narrow orifice.

This is hung over the edge of a vessel with its short arm external. The vessel must be of such dimensions that, when the liquid inside is level with the outer end of the siphon, there is a sufficient depth to submerge the solid.

The vessel is then filled with the liquid to slightly above this level, the siphon filled by suction, and as much liquid as will is allowed to flow out into a beaker.

The flow having ceased, the solid is gently dropped into the vessel, and the liquid which flows from the siphon collected in a measuring-glass previously placed to collect it.

The siphon reduces the level of the liquid to that of its outer end. This condition exists before the solid is immersed,

and finally when all flow has ceased. Consequently the volume of liquid in the measuring-glass is the volume raised above this level by the immersion, and equals that of the solid.

The density may also be determined if the solid be previously weighed.

The most convenient liquid for use is water or alcohol.

12.

To determine the Density of a Body which is denser than Water, and insoluble in it.—The density of a substance is, correctly speaking, its mass per unit volume. The term, however, is often applied to the weight of the same volume. With this latter significance, it is numerically equal to the specific gravity of the substance, *i.e.* the ratio of the mass of a volume of the substance to that of the same volume of water at 4° C.

Though "specific gravity" is an expression established by long usage, it is preferable to use the term "specific density."

It will be noted by those acquainted with dimensional equations that, whereas specific density is of zero dimensions, density is of the form $[M]\ [L]^{-3}$.

The specific density is that found experimentally, but from this the absolute density may be calculated.

A jar is placed over the left-hand pan of a balance upon a stool so adjusted that the pan can move freely beneath.

A length of fine thread is measured, a loop made at one end, and its weight found by suspending it from the hook supporting the pan. The free end is then tied to the body which is suspended inside the jar. Loose ends of the thread should be cut away and measured, so that the actual length in use is known. Let L be the original measured length, l the total length cut away, w_1 the original weight, w the final weight. Then—

$$w = w_1 \frac{L - l}{L}$$

Very fine wire may be advantageously employed instead of thread, the same measurements being made.

The suspended body and thread are then weighed in the empty jar. Let their weight be W_1. Then $W_1 - w$ is the body's weight. Water recently boiled and cooled down is then poured into the jar until the body is completely covered. The apparent weight is found again. Let this be W_2. Then $W_2 - w$ is that of the body if the small volume displaced by the thread be neglected. Before this weighing, all air-bubbles should be removed from the solid by touching them with a wire or camel-hair brush. The difference between these weighings equals the weight of water displaced by the body.

Therefore the relative density to water at the existing temperature is—

$$\frac{W_1 - w}{(W_1 - w) - (W_2 - w)} = \frac{W_1 - w}{W_1 - W_2}$$

and to water at 4° C.—

$$\frac{W_1 - w}{W_1 - W_2} \times \text{density of water at the existing temperature}$$

Now, $W_1 - W_2$ is the approximate volume of the solid, and when divided by the density of water at the temperature of that used, is that volume exactly.

Dividing the body's weight by its volume, therefore, the density is found.

In exact determinations correction should be made for the displacement of air.

13.

To determine the Density of a Body soluble in Water.— The same method is followed as in the previous determination; but a liquid of known density, in which the body is insoluble, is used instead of water. Let this liquid's density be d. Then the volume of liquid displaced is equal to—

$$\frac{(W_1 - w) - (W_2 - w)}{d} = \frac{W_1 - W_2}{d}.$$

and this equals also the volume of the body.

Hence the density may be found.

For exact determinations it is necessary to know at what temperature the liquid has the density of d, and either to use it at this temperature, or apply corrections.

14.

To determine the Density of a Body less Dense than Water.—If the body, though lighter than water, be heavier than some convenient liquid, the method of the preceding determination may be applied.

The more generally applicable method is to find the weight of the body in air when suspended, as described in Art. 12. Let this be W_1. A heavier body, called a "sinker," of sufficient mass to sink the lighter, is then attached to the latter. Let the weight of the combination in air be W_2, and in water W_3.

Lastly, the "sinker" alone is weighed in water. Let its weight be W_4. For accuracy the weight of thread should in each case be deducted from the observed weights. It is an advantage for the sinker to be provided with a hook, so that a considerable amount of immersed thread may be avoided.

The weight of water displaced by the body is—

$$(W_2 - W_3) - \{(W_2 - W_1) - W_4\} = W_1 + W_4 - W_3$$

From this it will be observed that so far as obtaining the result is concerned, the weighing of the combination in air may be omitted.

The relative density to the water used is—

$$\frac{W}{W_1 + W_4 - W_3}$$

and to water at 4° C.-

$$\frac{W_1}{W_1 + W_4 - W_3} \times \text{density of water at existing temperature}$$

Now, $W_1 + W_4 - W_3$ is approximately, and $W_1 + W_4 - W_3$

divided by the density of water at the existing temperature is exactly, the volume of the body.

Therefore, dividing the weight of the body by its volume, the density is found.

Results should be tabulated thus:

(*a*) Weight of body in air.
(*b*) Weight of combination in water.
(*c*) Weight of sinker in water.
(*d*) Volume of body.
(*e*) Density of body.

15.

To determine the Proportions in which Two Substances of Different known Densities enter into the Composition of a Solid.—Let d_1 and d_2 be the densities of the two constituents respectively, V_1 and V_2 their volumes, and let no change of volume occur on mixing.

The compound body is weighed in air and water as explained above. Let W be its weight, and V its volume. Then, the total weight of the body being the sum of the weights of its components—

$$W = V_1 d_1 + (V - V_1) d_2$$

$$V_1 = \frac{W - V d_2}{d_1 - d_2}$$

$$\text{and } V_2 = \frac{W - V_1 d_1}{d_2}$$

In this way the volumes are determined.

If the solid contain air cavities, the total volumes of the latter may be found with sufficient accuracy in most cases by putting d_1 or $d_2 = 0$.

16.

To determine the Density of a Liquid—*First Method.*—A body is suspended by a thread from the hook on which the

scale-pan hangs and weighed in air and in water. Let the difference between these weighings be W_1.

The jar of water is then removed, the body dried, and a jar of the liquid whose density is required is substituted. The body is now weighed when immersed in the liquid. Let the difference between the weights of the body in air and in the liquid be W_2.

Now, W_1 and W_2 are the weights of water and of the liquid which have the same volume as the body.

The relative density of the liquid to water at the existing temperature is—

$$\frac{W_2}{W_1}$$

and to water at 4° C.-

$$\frac{W_2}{W_1} \times \text{density of water at existing temperature}$$

This expression is also numerically equal to the density of the liquid.

Second Method.—By means of a **density bottle.** The density bottle is a small glass flask having a capacity of 10 to 100 grams of water, the larger sizes furnishing more accurate results. The stopper is in some cases drilled through, allowing any displaced liquid to escape when it is inserted in the bottle. In other cases, a thermometer with its bulb inside the bottle, and its scale rising above the stopper, is made in one piece with the latter. Sometimes a side tube is present from which the excess of liquid may escape. This generally bears a mark to which the level is adjusted.

The bottle is placed in the balance-pan, and weighed when empty and perfectly dry. It is then filled with water, which should have been boiled some time previously to expel any gases in solution, and after being thoroughly dried externally is again weighed.

The difference of these weighings is the weight of water contained. Let it be W_1.

The water is then poured away from the bottle, as much being shaken out as possible. The bottle should then be rinsed with alcohol, and dried by means of small rolls of filter or blotting-paper, ultimately being washed out with ether.

When perfectly dry it is filled with the liquid of which the density is required, and again weighed. Let the difference between this weight and that of the empty bottle be W_2.

Then, since W_1 and W_2 are the weights of equal volumes of water and the liquid, the relative density of the latter is $\frac{W_2}{W_1}$; or, applying the correction for temperature, which should be observed at the time of weighing, to water at 4° C.—

$$\frac{W_2}{W_1} \times \text{density of water at observed temperature}$$

This is numerically equal to the density of the liquid.

A **Sprengel's tube** is sometimes used instead of the density bottle. This is a tube doubled on itself like a U-tube, the open ends being turned outwards away from one another and drawn out to narrow orifices. It is filled by suction, and dried by drawing alcohol and afterwards air through. Its method of use is, otherwise, as that of the density bottle. In the absence of both, a piece of glass tubing may be blown into a necked bulb, and a mark, to which it should be filled, made on the neck.

17.

To determine the Density of a finely divided Solid by Means of the Density Bottle.—The weight of the perfectly dry and empty bottle is firstly found. Let this be w.

The fine solid is then put in the bottle, and the combined weight found. Let this be W.

The weight of the solid in air is then $W - w$.

Sufficient distilled water is poured in to cover the substance, and air-bubbles are removed by rotating the bottle about its axis while held in an inclined position. It is subsequently

filled with the water, dried externally, and again weighed. Let its weight now be W_1.

The solid is thoroughly washed out, and the bottle weighed when filled with water alone. Let this weight be W_2.

The weight of water displaced by the solid is then—

$$(W_2 + W - w) - W_1$$

and the relative density of the solid to water at the existing temperature is—

$$\frac{W - w}{W_2 + W - W_1 - w}$$

and to water at 4° C.—

$$\frac{W - w}{W_2 + W - W_1 - w} \times \text{density of water at existing temperature}$$

This is numerically equal to the density of the solid.

If the substance be soluble in water some liquid of known density in which it is insoluble may be employed in place of the latter.

Let the density of the liquid be d.

Then the volume of the substance equals $\dfrac{W_2 + W - W_1 - w}{d}$,

and its density is $\dfrac{(W - w)d}{W_2 + W - W_1 - w}$.

In making an exact determination, it is necessary to know at what temperature the liquid has the density d, and to use it at this temperature or apply corrections.

18.

To determine the Density of a Solid by Means of Nicholson's Hydrometer.—This instrument consists of a hollow float made of thin sheet metal, from the top of which rises a thin stem carrying a pan; another wire projects from the bottom, and is connected to a weighted pan below.

The centre of gravity, preferably called the centre of mass,

is near its lower end; so that if floating it maintains a vertical position.

Before use the upper wire should be rubbed with a piece of rag or cotton wool moistened with alcohol. This removes any greasiness, which if present creates an uncertainty in the level at which the hydrometer floats.

A tall jar is then filled with water which has been boiled some time previously and cooled.

In this the hydrometer is floated, any adhering air-bubbles being removed by a brush, and weights are placed in the upper pan until the mark on the upper stem is level with the surface of the water. If no available weights exactly do this, the weights which respectively allow it to rise above, and sink it below, the surface should be noted, and that which would be requisite estimated. Let this be W_1.

The weights are then removed, the solid placed in the pan, and sufficient weights added to again bring the mark to the surface. Let these weights be W_2. The weight of the solid in air is then $W_1 - W_2$.

The body is then removed to the lower pan, and weights placed in the upper till the mark is as before. Let these be W_3. The weight of the immersed solid is $W_1 - W_3$.

The difference of these weights in air and water, viz. $W_3 - W_2$, is the weight of the solid's volume of water.

Hence the relative density of the solid to water at the temperature of that used, which should be noted, is—

$$\frac{W_1 - W_2}{W_3 - W_2}$$

and to water at 4° C.—

$$\frac{W_1 - W_2}{W_3 - W_2} \times \text{density of water at the existing temperature}$$

This is also the density of the solid.

If the solid be soluble in water some other liquid in which it is insoluble and of which the density is known may be used. The corrections in such a case have been pointed out in Art. 17.

19.

To determine the Density of a Liquid by Means of Nicholson's Hydrometer.—The hydrometer, being clean and dry, is firstly weighed. Let its weight be w. It is then floated in water previously freed from dissolved gases by boiling, and, any adhering air-bubbles being removed, the weight which, placed in the upper pan, brings the stem mark to the surface of the water is noted. Let this be W_1. Then $W_1 + w$ is the weight of water which equals in volume that portion of the instrument immersed.

The hydrometer is taken from the water and carefully dried. The jar also is emptied, dried, and filled with the liquid of which the density is to be found. In this, with the same precautions as before, the hydrometer is floated, and the weight which, placed in the upper pan, brings the stem mark to the surface observed. Let this be W_2. Then the weight of liquid displaced is $W_2 + w$. Thus $W_1 + w$ and $W_2 + w$ are the weights of water and of the liquid respectively having the same volume.

The relative density of the liquid to water at the temperature of that used is—

$$\frac{W_2 + w}{W_1 + w}$$

and to water at 4° C.—

$$\frac{W_2 + w}{W_1 + w} \times \text{density of water at the existing temperature}$$

This is also numerically equal to the density of the liquid.

20.

To determine the Density of a Liquid by Means of a Variable Immersion Hydrometer.—Since a body when floating is immersed to such a depth that the weight of the volume of liquid displaced equals its own weight, it is obvious that in

liquids of different densities different volumes of the body will be immersed. It is therefore possible to graduate a conveniently shaped body so that particular divisions upon it are at the surface when it floats in liquids of certain densities. Such instruments are termed variable immersion hydrometers, and consist of a float, at the bottom of which is a heavy mass, and above which rises a graduated stem, indicating the density of the liquid in which it is floating by that reading which coincides with the surface.

One of the best-known forms is the "Twaddell's" hydrometer.

It is used for liquids denser than water, and, to obtain greater sensitiveness, any one instrument is graduated to show a difference of only about 0·13 in density between its lowest and highest readings; an entire set of six measuring densities between 1 and 1·85. To obtain the density the hydrometer reading is multiplied by 5, divided by 1000, and increased by 1.

Using an instrument for densities between 1 and 1·12, a series of solutions of common salt should be made, the weight of salt added in each case being noted, and the corresponding density of solution read from the hydrometer.

A hydrometer with an ungraduated stem may also be used to find densities if its stem be of uniform cross-section. In this case the diameter of the stem should be measured with a micrometer gauge or vernier calipers, and its sectional area calculated. Let this be A. The instrument should then be floated in water, and that point of the stem which is level with the surface noted. To do this the length of stem from the top down to the water-level must be measured, either by a cathetometer or by a scale held adjacently.

It is then floated in the liquid whose density is required, and the same observation is made as before. Let the length of stem between the noted points be L.

The hydrometer is then carefully dried and weighed. Let its weight be W.

Then the volume of the part immersed when in water was equal numerically to W, and when in the liquid to W ± AL,

and the weight of these volumes was in each case equal to that, W, of the hydrometer.

Therefore the density of the liquid is $\frac{W}{W \pm AL}$

Owing to the difficulty of determining the positions on the stem with exactness, and the density found thus being only approximate, it is no particular use applying a temperature correction for the density of the water.

21.

To determine the Density of a Liquid by the Method of Balancing Columns.—The apparatus required for this method may be in either of two forms.

(*a*) Two glass tubes, not less than half a metre long and about 7 mm. bore, are clamped vertically and adjusted by a plumb-line, one dipping nearly to the bottom of a glass vessel of water, the other in one of the liquid for which the density is required. The upper end of each tube is connected by a short length of rubber tubing to a three-way piece.

The remaining branch of the three-way piece is also furnished with rubber tubing, which may be closed by a clip. Air is sucked from this tube, drawing up a column of liquid in each limb.

When that of the less dense liquid is near the top of its tube, the clip is tightened. The heights of the columns above the external surfaces are then measured by the cathetometer or with a scale.

Now, in each tube the pressure of the column per square centimetre at the level of the external liquid equals the atmospheric pressure minus the pressure of the enclosed air.

Thus, if H_1 be the height of the water-column, H_2 that of the liquid, d_1 the density of the water, d_2 of the liquid—

$$H_1 d_1 = H_2 d_2, \text{ and } d_2 = \frac{H_1}{H_2} d_1$$

and the density may be determined.

(b) This form of apparatus consists of two glass tubes of 7 mm. bore, bent round at one end forming U-tubes, of which the one limb should be about 15 cms. long, the other not less than half a metre. These should be clamped vertically and adjusted by a plumb-line. Their shorter limbs are connected by a short piece of rubber tube well bound to the glass tubes. Water is poured into the one, the liquid into the other tube, imprisoning a quantity of air in the short limbs. Water and liquid are then alternately poured into the tubes until one of them is nearly full. During this filling, care should be taken that the levels in the short limbs do not reach the indiarubber tube.

The difference of levels in the limbs of the water-tube and of the liquid's tube are measured, and, as before—

$$H_1 d_1 = H_2 d_2, \text{ and } d_2 = \frac{H_1}{H_2} d_1$$

It will be noted that, by either method, the accuracy of the determination is proportional to the lengths of the columns.

22.

To determine the Density of Water at Various Temperatures by Means of a Hydrometer.—For this purpose a special form of Nicholson's hydrometer should be used. Sensitiveness in such an instrument is proportional to the volume of the float, and inversely as the square of the stem's diameter. Therefore one having a slender wire surmounted by a small pan should be adopted. The size of the float is limited by the fact that if the mass of the instrument be large its period of oscillation is inconveniently large.

An instrument of blown glass, weighted below so that its stem floats vertically, is far superior to one of metal. Dipping the open end of the stem into a little Indian ink for about 2 cms. of its length, so that its interior is blackened, forms an excellent adjusting mark.

The method of use is as explained in Art. 19.

The water to be used should be poured while hot into the jar. The weight which brings the stem-mark level with the surface and the temperature having been noted, the water should be allowed to cool a few degrees, and the observations repeated. As lower temperatures than those usual in laboratories will be required, it is well to use as small a jar as possible, and to place it inside a larger vessel, which may act as a cooling jacket. By putting cool water, and ultimately ice, in this vessel, the water in the jar, which should be stirred between the observations, may be brought almost to zero. Readings should be very carefully taken near 4° C.

Results should be tabulated thus:

Temperature.	Weights in pan.	Total weight.	Density.

A curve should be plotted with temperatures as abscissæ, and the difference between the density at those temperatures and the maximum density as ordinates.

23.

To determine the Density of a Solid by Means of Jolly's Balance.—This instrument consists of a spiral spring fixed above to a rigid frame, and carrying two pans below, one vertically beneath the other. On the straight wire proceeding from the spring to the upper pan is a mark, the position of which may be read by a cathetometer or a scale affixed to the frame. This mark should pass completely round the wire, and the scale be engraved on mirror glass, so that upon reading, the mark and its image may be made to coincide, and parallax avoided.

In making a determination the lower pan is immersed in water to such a depth that the arms connecting it to the single vertical wire are entirely submerged. The solid is then placed in the upper pan, and the division of the scale at which the

mark comes to rest is noted. Weights are now substituted for the solid, and adjusted until the mark is in its previous position. This weight equals the weight of the solid in air. The weights being removed from the upper pan, the solid is placed in the lower, and carefully freed from any adhering air. Weights are now added to the upper pan until the mark is again at the noted scale division. This weight equals that of the water displaced by the solid.

The relative density of the solid to the water used is its weight divided by that of the water. Multiplying this by the density of water at the existing temperature, the relative density to water at 4° C. is found. This is equal to the density.

If the solid be soluble in water, some liquid in which it is insoluble, and for which the density is known, may be used. In this case the expression given above for the density must be multiplied by the density of the liquid.

For accurate work it is necessary to know at what temperature the liquid has that density, and either to use it at that temperature or apply corrections.

24.

To determine the Density of a Liquid by Jolly's Balance.—The lower pan is immersed in water, and the weight of water displaced by some solid which is insoluble, found as explained. Let this be W_1.

The water is then removed, after which the solid, pan, and wires are carefully dried.

The operations are now repeated with the liquid in place of the water. Let the weight of liquid displaced be W_2.

The relative density of the liquid to water at the temperature of that used is—

$$\frac{W_2}{W_1}$$

and to water at 4° C.—

$$\frac{W_2}{W_1} \times \text{density of water at the existing temperature.}$$

This is also numerically equal to the density of the liquid.

25.

On the Use of the Hicks Bead Hydrometer.—In this instrument, which is extensively used for indicating the concentration of acid in storage cells, four coloured glass beads are enclosed in a flat glass tube perforated at the bottom.

Each of these beads is adjusted so that it just rises to the surface when in liquid of the density to which it corresponds. These densities are given on one side of the tube, viz. 1·2, 1·19, 1·17, and 1·15.

Unless the density of the liquid in which the tube is immersed happens to be equal to one of these, all that can be determined directly are two densities between which it is.

The density may, however, be found exactly, if there be a liquid with which the first liquid will mingle, or a solid which is soluble in it, provided that its density be not the same, and that no chemical action occurs.

Let a known volume, V_1, of the liquid whose density d_1 is required be taken, and the beads be placed in it. A measuring-glass is then filled with a noted volume of the other liquid for which the density d_2 is known.

This liquid is slowly poured and stirred into the first until one of the beads at the bottom begins to rise, or till one at the top begins to sink, according as the added liquid is more or less dense than the other. A density d_3 is thus indicated for the mixture. Let V_2 be the volume of liquid added. Then—

$$V_1 d_1 + V_2 d_2 = (V_1 + V_2) d_3$$

$$d_1 = \frac{(V_1 + V_2) d_3 - V_2 d_2}{V_1}$$

For practical convenience the densities d_1 and d_2 should differ considerably.

Except in the case of an aqueous solution, which can be diluted with water, other methods of determining the density present less trouble.

26.

To verify the Laws of Motion by Means of Atwood's Machine.—Atwood's machine consists of a vertical column some 8 feet or so in height, and graduated along its length. At the top is a pulley, which should have as little mass as is consistent with due strength.

The axle of the pulley should be carried by friction wheels to minimize friction. Over its grooved edge runs a fine cord, to each end of which weights may be attached. Three shelves, the vertical position of which is adjustable, are attached to the column, from which their relative positions may be read. These are vertically beneath the cord hanging from one side of the pulley. The middle shelf is cut away, so that certain weights can pass it uninterruptedly as the cord to which they are attached descends, and often takes the form of a ring.

Certain other weights, which may be termed "motive weights," are intercepted and lifted from the cord.

A seconds pendulum is sometimes attached to the column to facilitate the measurements of time. In the absence of this a stop-watch should be used.

The cord used, preferably one of silk, should be light, strong, and as inextensible as possible.

Two corrections have to be applied to the weights and masses employed: (*a*) For *friction*, the least weight which when added to one of the two suspended masses just maintains uniform motion should be found by adding it and gently starting motion: let this be w. (*b*) For the *inertia* of the pulley, which virtually increases the masses moved. Let the equivalent addition of mass be $\frac{w'}{g}$.

(1) To find the Acceleration produced with Given Masses at the Ends of the Cord, and a Given Motive Weight.—Let a weight W_1 be suspended from each end of the cord, and a motive weight W_2 be added to that end which is on the same side as the shelves. The weights on this side should rest upon

the uppermost shelf near the top of the column, and should be suddenly allowed to descend by turning the shelf on a hinge from beneath them. If this shelf be absent, the weights may be supported and released by hand. In either case the position of starting is noted.

The middle shelf is then adjusted until some exact period of time elapses, as indicated by the pendulum or stop-watch, between starting the motion and hearing the motive weight strike it. Let the time of falling be T_1.

The two upper shelves retaining the same position, the lowest is now adjusted until some exact interval of time elapses between hearing the weights strike the middle and bottom shelves. Let this be T_2.

It may be assumed that the velocity during this latter period has been uniform. The assumption is easily tested by reducing the distance by half, or doubling it, and noting the time of descent.

Let L_1 be the distance between the upper shelves, L_2 between the lower.

Then the velocity acquired in falling from the top to the middle shelf was $\frac{L_2}{T_2}$. And the time in which this velocity was acquired was T_1. Therefore acceleration produced is $\frac{L_2}{T_1 T_2}$.

(2) To determine "g," the Acceleration due to Gravity.—All bodies are attracted toward other bodies with a force proportional to the products of their masses, and inversely to the squares of the distances they are apart. This force is sufficient to draw surrounding bodies toward the centre of the earth until they reach its surface. Since this force is proportional to mass, and the acceleration of a body equals the applied force divided by its mass, the acceleration is independent of mass.

To find g, the operations described above are performed. Now, the force producing acceleration is $W_2 - w$, and the total mass moved is—

$$\frac{W_2 + 2W_1 + w'}{g}$$

or, omitting correction for the pulley—

$$\frac{W_2 + 2W_1}{g}$$

Hence, in the latter case—

$$\frac{g(W_2 - w)}{W_2 + 2W_1} = \frac{L_2}{T_1T_2}$$

and g may be found.

To use the more accurate expression, the observations must be repeated, using a different motive weight.

If time is read by a stop-watch, the positions of the shelves should remain unchanged, but the new times be noted. Let W_2' be the motive weight, T_1' and T_2' the times. Then—

$$\frac{(W_2' - w)g}{W_2' + 2W_1 + w'} = \frac{L_2}{T_1'T_2'}$$

$$W_2' + 2W_1 + w' = \frac{gT_1'T_2'(W_2' - w)}{L_2}$$

Similarly, from the first set of observations—

$$W_2 + 2W_1 + w' = \frac{gT_1T_2(W_2 - w)}{L_2}$$

Therefore—

$$g = \frac{(W_2 - W_2')L_2}{T_1T_2(W_2 - w) - T_1'T_2'(W_2' - w)}$$

If, however, the time be taken by a pendulum, it is better to readjust the shelves so that, with the changed motive weight, the times are as before. Let L_2' be the new length between the middle and lower shelf, W_2' the new motive weight.

Then $\dfrac{(W_2' - w)g}{W_2' + 2W_1 + w} = \dfrac{L_2'}{T_1T_2}$

and $\dfrac{(W_2 - w)g}{W_2 + 2W_1 + w} = \dfrac{L_2}{T_1T_2}$

$$\text{hence } g = \frac{(W_2 - W_2')}{T_1T_2\left(\dfrac{W_2 - w}{L_2} - \dfrac{W_2' - w}{L_2'}\right)}$$

(3) To determine the Connection between Time and Distance moved, when a Body starts from Rest and moves under Uniform Acceleration.—Equal masses should be hung at the ends of the cord, and a small motive weight added to one of them.

The distance between the top and middle shelf should be adjusted so that it is travelled over in half a second.

It should then be increased to give times of 1, 1·5, 2, etc., seconds. A curve should be plotted with the distances as abscissæ, and the time squared as ordinates.

(4) To determine the Connection between Final Velocity and the Distance moved from Rest under Uniform Acceleration.—The distance between the upper and middle shelves should be at first small, and the distance between the middle and lowest adjusted so that it is passed over in some exact time. This distance divided by the time is the velocity when acceleration ceases at the middle shelf.

This velocity should be found thus for various distances between the top and middle shelf.

Results should be entered thus:

Distance moved under acceleration.	Distance moved with uniform velocity.	Time.	Velocity acquired during acceleration.

A curve should be plotted with distances moved under acceleration as abscissæ, final velocities squared as ordinates.

27.

To determine the Relation between the Period and Length of a Simple Pendulum, and by its means to find the Value of "g," the Acceleration due to Gravity.—The ideal simple pendulum consists of a mass concentrated in a point,

and suspended by a thread having no mass, so that determinations depending on formulæ deduced from the theoretical case can only be approximate. A small heavy sphere attached to a silk thread, however, gives results which, when corrections have been applied, are highly approximate.

Let O be the point of suspension of a simple pendulum, A the position of the bob's centre of mass when at rest, B its position at any instant during a vibration. The forces acting at B are the tension of the thread along OB, the force of gravity parallel to OA, and that producing an acceleration toward A along AB.

These forces are proportional to the sides of the triangle OAB. Therefore acceleration toward A is $g\frac{AB}{OA}$, and if AB equal unity, this becomes $\frac{g}{L}$, where L is the length of the pendulum. If the angle AOB be small, AB may be considered at right angles to OA.

Now, it is proved in works on dynamics that the period of a body executing simple harmonic motion is equal to 2π divided by the square root of the acceleration acting at unit distance from the mean position. A body has simple harmonic motion when it oscillates in a straight line on either side of a mean position, its acceleration being always directed towards the mean position, and proportional to its distance from it.

It will be noted that the pendulum's motion, under the approximations mentioned, is of this type. Therefore its period, *i.e.* the interval of time between two successive passages in the same direction across the mean position, is given by—

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The fact of the bob having volume may be corrected for by applying the formula, which is elsewhere shown to give the period of a compound pendulum, viz.—

$$T = 2\pi\sqrt{\frac{(k^2 + L^2)}{gL}}$$

where k is the radius of gyration of the spherical bob about its own axis. For a sphere, $k^2 = \frac{2r^2}{5}$.

The fact of the path being an arc, and not exactly a straight line as assumed, may be partially corrected for by multiplying the above by $\left(1 + \frac{AB^2}{16L^2}\right)$, where AB is the mean amplitude of the vibrations noted. This, however, if various lengths are used, is a troublesome correction, and may generally be neglected.

To determine the relation between the period and length, the thread should be attached to a perfectly rigid point of suspension, its length from there to the top of the bob measured, and the diameter of the latter found. The length of the pendulum is then the length of thread plus the radius of the bob. The number of swings is noted by a telescope focussed on the thread when at rest; if this be on the upper part of the thread, the same adjustment serves for shorter lengths. In the absence of a telescope, a piece of paper with a line upon it, and another with a narrow window about an eighth of an inch wide cut out, should be placed behind and before the thread respectively in such a way that the centre of the slit, the thread, and the line coincide.

The bob should then be drawn slightly to one side and released, so that it swings parallel to the paper screen. In a very exact determination, a thread should be looped round the bob and drawn back. Swinging is then started by burning this near to the bob.

The amplitude should be but a few centimetres. At the exact instant that, viewed through the slit, the thread is seen to coincide with the line, a stop-watch, previously set to its zero, should be started. Counting the next transit in the same direction as 1, the watch should be stopped at the twentieth. Care must be taken to count only those passing in the same direction. The time of the twenty swings is then observed. Let this be T. The period would then apparently be $\frac{T}{20}$, but a stop-watch, as a rule, only indicates to a fifth of a second, so

that the actual time may have been only a very little short of $T + \frac{1}{5}$ secs. It can only be said, therefore, that the period is between $\frac{T}{20}$ and $\frac{T}{20} + 0{\cdot}01$ secs.

The total time of a large number of vibrations should now be observed, the watch being started and stopped at transits in the same direction. The number in the interval need not be counted. Let T′ be this time, which, since it is the time of a large number of complete swings, must be divisible by the correct period. Then, if the integer, which differs least from the quotients of $\dfrac{T'}{\frac{T}{20}}$ and $\dfrac{T'}{\frac{T}{20}+0{\cdot}01}$, be the same in each case, this integer is the number of swings in the time T′.

If the nearest integers differ, shorter times must be taken for T′, until they become identical.

Let this integer be N. Then, since there are N swings in the time T′, $\frac{T'}{N}$ is the period; or, since the watch only reads accurately to one-fifth of a second—

$$\text{Period} = \frac{T' + 0{\cdot}1}{N} \text{ to within } \frac{0{\cdot}1}{N} \text{ secs.}$$

Hence, using the correction for the bob's volume, g should be found.

Various lengths between 100 and 30 cms. should be taken, and the periods found.

Results should be tabulated thus:

T.	T′.	N.	T′ ÷ N.	Period.

A curve should be plotted with lengths as abscissæ, and periods squared as ordinates.

28.

To determine the Length of a Seconds Pendulum by Means of Kater's Pendulum.—The Kater's pendulum is an instrument devised to find the length of a seconds pendulum more accurately than is possible using the simple pendulum.

A simplified form consists of a bar, weighted at one end, and carrying two sets of knife-edges. One pair of these is movable along the bar, which is so graduated that the distances between the upper and lower pairs may be read.

A still less elaborate modification is a long bar drilled with holes at short intervals along its length, and swung from a knife-edge passing through one of the holes.

Before entering into details as to its use, reference should be made to Art. 34, in which the vibrations of a compound pendulum are discussed.

It is there shown that if the pendulum oscillate about either pair of knife-edges with the same period, the distance between these edges is equal to the length of a simple pendulum having that period.

The arrangements having been made as described in Art. 27 for noting the transits, and a thin straw having been affixed to the pendulum, in line with its axis, to serve as a definite line for this purpose, the period is found, as there explained, when the pendulum swings on its fixed edges.

The period is then found about the movable edges, which are adjusted until it is the same about themselves and about those that are fixed. Diminishing the distance between the knife-edges shortens the period about those that are movable.

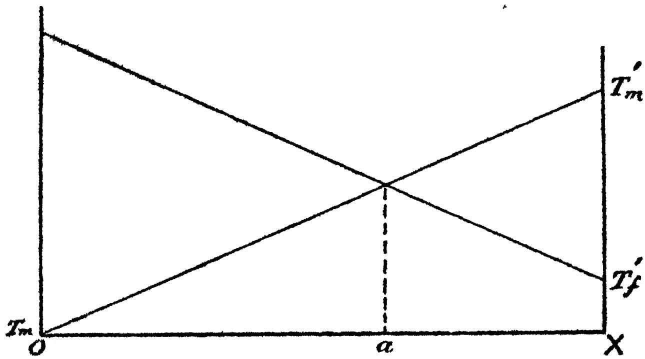
In the case of a simple bar, three holes, two of which are adjacent, should be found. These must be such that when swung from the adjacent holes, the periods are respectively shorter and longer than when swung from the third. The following method, which is in reference to the form with knife-edges, must then be applied to find the correct position:—

Let two near positions of the movable edges be found such

that the periods are respectively greater and less than about the fixed edges.

Let T_f and T_m be the periods about the fixed and movable edges when that of the latter is least, T'_f and T'_m when that of the former is least.

These should be plotted as shown, where OX represents the distance between the shorter and longer positions of the



movable edges. The ordinates represent the four periods minus that which is least.

The point of intersection of the lines gives the correct position a of the edges; and, by adding its ordinate to that period which was deducted, the corresponding period is obtained.

The length of a seconds pendulum, *i.e.* one of 2 seconds' period, should then be calculated.

29.

To construct a Blackburn's Pendulum of 1-second and ½-second Periods.—This is a simple pendulum so arranged that while, as a whole, it vibrates in one plane, some length of it vibrates in another at right angles independently, the actual motion of the bob being compounded of the two vibrations.

As shown, the suspension is divided into two parts, the lower a single thread attached to the bob, and of such length

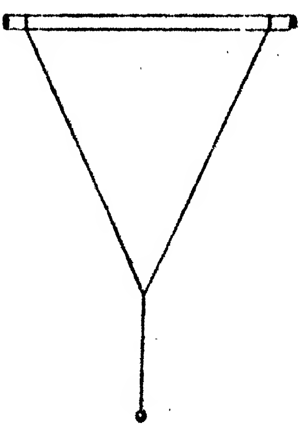
as to have the smaller period, the upper looped, and of such length that the vertical distance between the points of suspension and the centre of the bob is that of a pendulum having the greater period.

The length of a pendulum of 1 second's period being calculated, that for one of half the period is readily found from the relation—

$$\frac{T^2}{T_1^2} = \frac{L}{L_1}$$

A thread is attached to the bob, and the shorter length marked off upon it from the centre of the bob. The free end is then tied round a thread of sufficient length to form the double suspension in such a way that the knot is at the marked point on the shorter, and can slide along the longer thread.

One end of the long thread is fastened to a horizontal bar, and the other temporarily tied to the same, the knot being slipped down to the lowest possible point on it. The vertical distance from the points of attachment on the bar to the centre of the bob is now adjusted until the period of swing is that required. This vertical height may conveniently be altered by shifting the points of attachment horizontally along the bar. These should be a considerable distance apart, as the maximum angle through which the single suspension can swing is equal to that between the threads above it, any greater displacement causing the latter to twist together.



The resultant paths of the bob are known as "Lissajous' figures," and vary in form according to the relative phase of the oscillations. They may be traced by substituting a funnel, with a narrow pipe, filled with sand, for the bob.

30.

To compare the Periods of Two Pendulums by the Method of Coincidences.—This method is applicable when the periods are approximately equal or multiples of each other.

The two pendulums are suspended so that when viewed through a slit and at rest they coincide, one suspension being in front of the other. A rough observation is then made of the ratio of the periods. Let it be found that the shorter oscillates about n times, while the other does so once.

Both are set swinging, and a stop-watch is started at the instant they cross the slit together in the same direction. The shorter gains on the other, crossing the slit earlier at first, and subsequently later. The transits of the longer after starting the watch are counted, until both again cross together in the same direction, when the watch is stopped. Let there be N transits in this interval T, that at which the watch was started being counted as 0. Then the period of the longer one is $\frac{T}{N}$, and that of the shorter $\frac{T}{nN \pm 1}$, the positive sign being taken if it gained on the other, the negative if it lagged.

If N, as found above, is a small number, the time of several coincidences should be taken, each coincidence being marked by a dash on a sheet of paper, while the eye is kept at the slit counting N. Let the watch be stopped at some coincident transit, which is found from the paper to be m, and let the vibrations of the longer pendulum and the total recorded time be N and T respectively.

Then $\frac{T}{N}$ is the longer period, and $\frac{T}{nN \pm m}$ the shorter.

31.

To determine "g," the Acceleration due to Gravity, by rolling or sliding a Body down an Inclined Plane.—If a body of mass m roll or slide for a distance l down an inclined plane

of height H and of slanting length L, it loses an amount of potential energy equal to $mgl\frac{H}{L}$. This energy has been expended in the following ways:—

(*a*) Overcoming friction between the body and the plane.
(*b*) Imparting kinetic energy of translation to the body.
(*c*) Imparting kinetic energy of rotation to the body.

In the case of the body entirely sliding, however, the last of these quantities is absent.

Supposing, as is usually the case, the plane is of constant length, but of adjustable height, the quantity (*a*) may be found by adjusting the height till the body moves down the plane with uniform velocity. If this height be h—

$$(a) = mgl\frac{h}{L}.$$

It is necessary to start the motion by a slight push, as the statical friction is somewhat greater than that when the body is in motion.

The remaining terms (*b*) and (*c*) are determined by increasing the height until the body starts without assistance when placed upon the plane, and observing the times in which it rolls over different distances, beginning with as short a distance as possible, and proceeding to the whole length of the plane by increasing steps. To do this, two blocks may be used, one supporting the body, and the other adjusted at a known distance. The body is released by drawing away the block, and a stop-watch started at the same instant. The watch is stopped when the roller is heard to strike the lower block. Three or more readings should, of course, be taken for each distance, and the mean time recorded.

Results should be tabulated thus:

Distance moved.	Time of motion.	L.	$H - h$	$s\left(\frac{lL}{t^2(H-h)}\right)$

Now, if v be the velocity of motion down the plane at the end of any time t—

$$v = \frac{2l}{t}$$

For the final is twice the mean velocity since the body started from rest. But the kinetic energy of translation equals $\frac{mv^2}{2}$. Hence—

$$(b) = \frac{2ml^2}{t^2}$$

If the body rotates, its kinetic energy of rotation is equal to $\frac{mk^2\omega^2}{2}$, where k is the radius of gyration and ω the angular velocity. The latter equals $\frac{v}{r}$, if r be the radius of the body, since $\frac{v}{2\pi r}$ revolutions are made per second. Hence—

$$(c) = \frac{mk^2v^2}{2r^2} = \frac{2mk^2l^2}{r^2t^2}$$

For a solid cylinder rotating about its axis, k^2 equals $\frac{r^2}{2}$; for a hollow cylinder of external radius r and internal r', k^2 equals $\frac{r^2 - r'^2}{2}$; and for a solid sphere, k^2 has the value $\frac{2r^2}{5}$. Finally, therefore—

$$mgl\frac{H}{L} = mgl\frac{h}{L} + \frac{2ml^2}{t^2} + \frac{2mk^2l^2}{r^2t^2}$$

$$g = 2\left(1 + \frac{k^2}{r^2}\right)\frac{Ll}{(H - h)t^2}$$

The mean value of $\frac{2Ll}{(H - h)t^2}$ should be found from the table and employed.

32.

To determine "g," the Acceleration due to Gravity, by Means of a Spiral Spring, and to find the Modulus of the Latter.—Provided that the load does not exceed a certain limit, the elongation of a spiral spring is proportional to it.

The ratio of the load, or stretching force, to the increase in length per unit length of the spring, is the latter's modulus of rigidity.

In determining it, the length of the unloaded spring should be measured, and a cathetometer focussed on its lower end.

Weights sufficient to produce observable elongations should then be hung on the spring, or placed in a suspended scale-pan, and increased by steps, the corresponding extensions being noted by the cathetometer. (Weights of 20, 30, 40, and 50 grams are suitable for a brass spring about 3 cms. diameter, 10 cms. long, and of 0·2 cm. diameter wire.) Readings should be made increasing and diminishing the load.

Results should be tabulated thus:

Original length	Load	Extension.	$\frac{\text{Extension}}{\text{Original length}}$	Modulus.

A curve should be plotted with loads as abscissæ, extensions as ordinates.

The above remarks apply to the positions of rest. When the load oscillates vertically, it executes simple harmonic vibrations, and the period of any such vibration is $2\pi\sqrt{\frac{1}{\text{acceleration}}}$, the acceleration being that at unit displacement from the mean position, which equals the force producing unit elongation divided by the mass moved. The latter equals the mass of the weight in the pan plus about half the mass of the spring itself. Let w be the weight producing unit extension.

A noted weight is placed in the pan, and the period of vibration found in the same manner as for a pendulum. Let this be t, and the weight plus half that of the spring be W. Then—

$$t = 2\pi\sqrt{\frac{1}{\frac{w}{W \div g}}} = 2\pi\sqrt{\frac{W}{wg}}$$

Hence g may be found.

33.

To find the Centre of Mass of a Plane Lamina.—It is assumed here that the lamina is of a substance through which pins may be driven to serve as suspensions, otherwise a thread must be attached by means applicable to the nature of the substance.

Let the lamina be suspended freely by a pin driven through near one edge, and from the pin let any small mass be hung by a thread in front. The point on the lower edge of the plate where the thread crosses is then carefully marked. The centre of mass is then somewhere upon the line drawn through the pinhole and this mark.

The pin is now driven through in a new position such that, as near as may be judged, the previous line will lie horizontally, and the mark made on the lower edge as before. Somewhere on the line drawn through these points the centre of mass will also lie.

Therefore it can only be at their point of intersection, and at the centre of the thickness, provided the density be uniform in that direction.

In finding the centre of mass for laminæ of certain shapes *e.g.* of an L form, where it lies outside, a drawing must be made on paper, adjusted to the suspended figure, and the observed points marked on it, after which the lines may be drawn.

34.

To determine the Moment of Inertia of a Body about any Given Axis.—The moment of inertia of a body about any axis is the sum of the masses of all the elements into which it may conceivably be divided, each multiplied by the square of its distance from that axis. It may be proved that the moment of inertia about any axis is equal to that about a parallel axis through the centre of mass, plus the mass of the body multiplied by the square of the distance between the axes.

To deduce the connection between the moment of inertia

E

and the period of swing, it is necessary to consider the oscillations of a compound pendulum. A compound pendulum may be defined as a body oscillating under the action of a force which is proportional to the distance of the body from its mean position, and always directed toward that point. When the body's mass is concentrated in a volume, small compared with the distance between its centre and that about which the body swings, the pendulum is called "simple." Strictly speaking, however, a material simple pendulum is unobtainable.

The point about which a body swings is termed its centre of suspension. That point on the line passing through the centres of mass and suspension at which the whole mass might be concentrated without altering the period or the kinetic energy, is called its centre of gyration, the distance between this centre and that of suspension being the radius of gyration, usually indicated by k. The moment of inertia, indicated by I, is consequently equal to Mk^2, if M be the mass.

Let a compound pendulum, at an angle θ with the vertical, fall to an angle θ', acquiring angular velocity ω, and having its centre of mass lowered a vertical distance H. Then—

$$\tfrac{1}{2}Mk^2\omega^2 = MgH$$

Let l be the distance between centres of mass and suspension; consequently—

$$\omega^2 = \frac{2gl}{k^2}(\cos\theta' - \cos\theta)$$

Now, in a simple pendulum k and the length are identical. Let L be the calculated length of one having the same period as the compound pendulum. Then, under the same conditions—

$$\omega^2 = \frac{2g}{L}(\cos\theta' - \cos\theta)$$

$$\text{whence } L = \frac{k^2}{l}$$

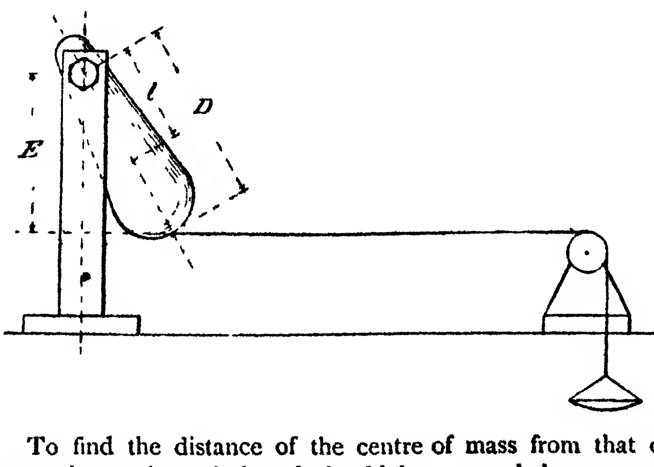
But the period is equal to $2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{k^2}{gl}} = 2\pi\sqrt{\frac{I}{Mgl}}$

Thus, by observing the body's period when oscillated about the given axis, and finding its centre of mass, I may be determined. Also—

$$\text{I about centre of mass} = \text{I} - \text{M}l^2$$

In the case of a plane lamina, the oscillation may be about a pin driven through, and the centre of mass found as explained in Art. 33, after which the plate is weighed to find M.

If the body be not plane, a frame containing two pointed screws, fixed coaxially at an adjustable distance apart, is required. The points are screwed to clip the extremities of the given axis between them, but allowing the body to swing. The period is then noted.



To find the distance of the centre of mass from that of suspension, a looped thread should be suspended over each point, close to the body, and a straight piece of wire suspended horizontally in the loops, so that it passes beneath the body. The latter being at rest, the wire is raised by drawing up the threads until it just touches it, when the point of contact should be marked.

The centre of mass then lies in the plane passing through this mark and the screw-points.

The vertical distance between the mark and the screw-points should be measured with a cathetometer. Let this be D.

A thread is then attached to the mark on the body, and led over a pulley held in a clamp about a metre distant. The body is then deflected by suspending a weight from the free end of the thread. This deflection may be about 30°. The height of the pulley is then adjusted until the thread is horizontal, and the vertical distance between its point of attachment and screw-points measured. Let this be E. Then, if M be as before the mass of the body, and W the weight on the thread, θ the angle of displacement—

$$WE = Mgl \sin \theta = Mgl(1 - \cos^2 \theta)^{\frac{1}{2}}$$

$$\text{hence } l = \frac{WED}{Mg(D^2 - E^2)^{\frac{1}{2}}}$$

The moment of inertia may also be determined by torsion, as explained in Art. 39, if the modulus of the wire be known.

35.

To determine the Moment of Inertia of a Wheel, and the Frictional Resistance Couple.—The wheel is rotated by a weighted cord, which unwinds itself off a drum attached to the spindle of the wheel. The end of this cord is looped over a short peg projecting from the drum, so that, when quite unwound, it falls freely away.

The potential energy lost by the weight in falling from rest through the height H is, if the weight be W, equal to HW.

The cord should be wound exactly N complete turns upon the surface of the drum, of which R represents the radius. Then—

$$HW = 2\pi RNW$$

Otherwise H may be directly measured by winding the cord, and taking the position of the weight when wound and

when on the point of slipping free. The weight W should be sufficiently large to start motion without assistance.

The potential energy lost is expended in the following ways:—

(*a*) Overcoming the friction in the bearings.
(*b*) Imparting kinetic energy to the wheel.
(*c*) Imparting kinetic energy to the falling weight.

The first of these quantities is equal to the frictional couple, *i.e.* that couple which would just maintain uniform rotation of the wheel against friction, multiplied by the angle through which the wheel turns between the starting and dropping away of the weight. Let F be the frictional couple. Then—

$$(a) = 2\pi NF$$

F may be determined experimentally by finding the least weight, which, during its descent, rotates the wheel, when once started, with uniform velocity. If this weight be w—

$$F = w \times R$$
$$\text{and } (a) = 2\pi NwR$$

Another method of determining F is to note how many revolutions, entire and fractional, the wheel makes between the time of a weight W falling off, and its coming to rest. If these be n—

$$2\pi nF = (b)$$

since the whole kinetic energy of the wheel is ultimately expended in work against F.

The kinetic energy of the wheel at the instant of the weights falling away is equal to—

$$\tfrac{1}{2}I\omega^2$$

if I be its moment of inertia, and ω its angular velocity.

Since it starts from rest, its final velocity is twice the mean during the period of its acceleration. To find this, the time T between the weight's starting and falling off is noted by a stop-watch. It is advisable to fix a block so that the weight strikes

it immediately after falling off, and to stop the watch when the knock is heard, or H may be taken just less than the distance to the floor. The mean angular velocity is then $\frac{2\pi N}{T}$, and—

$$\omega = \frac{4\pi N}{T}$$

$$\text{so that } (b) = \frac{8\pi^2 N^2 I}{T^2}$$

The energy in the falling weight W itself is equal to half its mass multiplied by its velocity squared. Its mean velocity during the descent is $\frac{H}{T} = \frac{2\pi NR}{T}$, and its final velocity when falling off is twice this. Therefore—

$$(c) = \frac{8W\pi^2 N^2 R^2}{gT^2}$$

Now—

$$HW = 2\pi RNW = 2\pi NF + \frac{8I\pi^2 N^2}{T^2} + \frac{8W\pi^2 N^2 R^2}{gT^2}$$

Hence, if the first method of determining F is adopted—

$$I = \frac{T^2}{4\pi N}\{R(W - w)\} - \frac{WR^2}{g}$$

or, if the second method is employed—

$$I = \frac{RWT^2}{4\pi N\left(\frac{n + N}{n}\right)} - \frac{WR^2}{g}$$

The determination may be made without any observation of time if a strip of smoked paper be fixed round the circumference of the wheel, and a tuning-fork with a style be adjusted to just touch it.

Let the period of the fork be t, and the measured diameter of the wheel be D.

The style of the fork is adjusted to touch lightly, and the fork bowed, immediately after which the weight is allowed to start its descent.

Shortly before the wheel has made a complete revolution, the fork is removed, and as soon as the wheel is stopped, the smoked strip is taken off. Two convenient lengths are then measured off on it in the ratios 1 and 4, each from the point at which the trace started, and the number of whole waves and any fractional part between the two measured points counted. Let these be *m*. Then the wheel turned from the starting-point to the first measured point in the time mt seconds. Let S be this distance as measured. The corresponding angle is $\frac{2S}{D}$.

$$\text{Now, the angle turned} = \tfrac{1}{2}\frac{\text{twisting moment}}{I}t^2$$

The height through which the weight falls when the wheel is turned this amount may be measured, and the loss of potential energy equated to the three terms as before.

It is necessary for the wheel to rotate with moderate rapidity to furnish a clear curve. This may be arranged by using a sufficiently heavy weight.

From the introductory remark in Art. **34**, it will be clear that by affixing bodies to the wheel, which in this case is more conveniently on a vertical axis with the cord over a second pulley, their moments of inertia may be determined.

36.

To determine "Y," Young's Modulus for the Material of a Given Wire.—If a substance be extended or compressed, it returns to its original form and volume upon removal of the force, provided it has not been strained beyond a certain extent, termed its "limit of elasticity." If strained beyond this limit, it acquires a "permanent set."

Young's modulus is a constant, applicable only within the limits of elasticity, and is the ratio of the force applied on unit area (the stress), to the change in length produced on unit length (the strain).

The simplest method of determining it is to suspend two

wires side by side from a rigid support, the one carrying a scale and just loaded to tautness; the other, on which investigation is to be made, carrying a vernier and a pan. The latter should have just sufficient load put in the pan to pull it straight, and the scale should then be read.

The load should now be increased by steps and the scale read for each. The same should, after the maximum load, be done in removing it by steps.

The length of the wire is then measured, the diameter found by the micrometer gauge, and the sectional area calculated.

Results should be tabulated thus:

Original length.	Load.	Scale reading.		Extension.		Mean extension / Original length
		Increasing load.	Diminishing load.	Increasing load.	Diminishing load.	

The load divided by the ratio in the last column is the modulus for the particular wire. This again, divided by the wire's sectional area, is Young's modulus for its material. The mean value should be taken as Y.

Curves should be plotted for the increasing and diminishing load results separately on the same axes, having loads as abscissæ and extensions as ordinates.

The volume elasticity is equal to $NY \div (9N - 3Y)$, where N is the modulus of rigidity.

37.

To determine "Y," Young's Modulus, by the Deflection of a Bar.—Let a bar, originally straight, be deflected into a curve, of which the mean radius is R. Then the part of radius less than R is in compression, and that of greater radius in tension, the length of the neutral layer of radius R being unaltered. Let the length of this layer be $\frac{2\pi R}{n}$. The

extension of any layer at radial distance l from the neutral layer is $\frac{2\pi l}{n}$, and if it be of unit width, and depth dl, the total force upon it, since $\frac{2\pi l}{n} = \frac{\text{force per unit area}}{\text{Y}} \times \frac{2\pi \text{R}}{n}$, is $\frac{\text{Y}}{\text{R}} l dl$. And the moment of this about the neutral axis is—

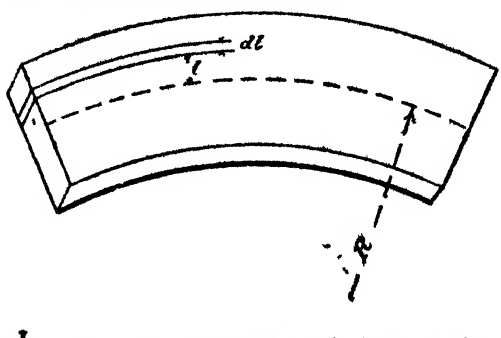
$$\frac{\text{Y}}{\text{R}} l^2 dl$$

The sum of these terms for all the layers in the bar is equal to the bending moment on it. Let this be M.

But $l^2 dl$ for the whole section is its moment of inertia about the axis.

$$\text{Therefore M} = \frac{\text{YI}}{\text{R}}$$

where I denotes the moment of inertia.



Now, $\frac{1}{\text{R}}$ measures the curvature of the beam, which may also be expressed by $\frac{d^2y}{dx^2}$, x being length along the bar, and y the deflection; so that—

$$\frac{d^2y}{dx^2} = \frac{\text{M}}{\text{YI}}$$

Let a weight W be attached to one end of a bar, the other end of which is fixed. Then the bending moment at the section distant x from the fixed end is $\text{W}(\text{L} - x)$; so that—

$$\frac{YI}{W} \cdot \frac{d^2y}{dx^2} = L - x$$

Integrating—

$$\frac{YI}{W} \cdot \frac{dy}{dx} = Lx - \frac{x^2}{2}$$

the added constant being o, since x and $\frac{dy}{dx}$ both equal o at the fixed end. Again integrating—

$$\frac{YI}{W}y = \frac{Lx^2}{2} - \frac{x^3}{6}$$

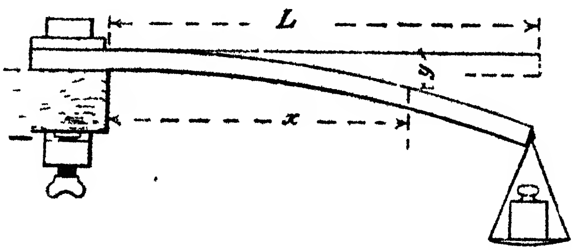
the constant again being o—

$$\text{Hence } y = \frac{W}{YI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)$$

and at the extreme end, where x and L are identical—

$$y = \frac{WL^3}{3YI}$$

I for a rectangular section $= \frac{\text{breadth} \times \text{depth}^3}{12}$; for a circular section, $\frac{\pi r^4}{4}$.



If the bar be supported at each end on knife-edges and loaded in the centre, the deflection is the same as would exist in one of half the length carrying half the load at its end.

$$\text{Hence its deflection} = \frac{\frac{1}{2}W(\frac{1}{2}L)^3}{3YI} = \frac{WL^3}{48YI}$$

The zero of the eye-piece scale of a cathetometer, or its cross-wire, should be focussed on some mark on, or connected to, the centre or end of the bar, according as the load is central or at the end. The load should then be increased by steps and, after the maximum, similarly diminished, the deflection being noted for each change. The length between the supporting edges is measured by a scale, the breadth and depth of the bar by calipers or a micrometer gauge.

Results should be tabulated thus:

Length of bar =
Breadth of bar =
Depth of bar =
I of bar's section =

Load.	Deflection.	Y.

The value of Y is given by $Y = \frac{WL^3}{3yI}$ for a bar fixed at one end, by $Y = \frac{WL^3}{48yI}$ for one supported at each end. The mean of the last column should be taken.

38.

To determine "Y," Young's Modulus, for Indiarubber by a Method of Oscillations.—One end of a rubber cord should be rigidly clamped so that it hangs vertically, and a light pan is attached to the lower end. The length of the cord is then measured. Let it be L. The period of oscillation is now found by placing a weight in the pan and observing the time of twenty or more vibrations. The motion being simple harmonic, its period is—

$$2\pi\sqrt{\frac{W}{wg}}$$

as given in Art. 27, where W represents the weight in the pan plus that of the pan and half that of the cord itself, w being the force extending the cord by unit length. But by the definition of Y, this force $w = \frac{Y}{L} \times$ sectional area of unloaded cord $= \frac{Y}{L}\pi r^2$, where r is the radius of the unloaded cord, found by measuring its diameter with a micrometer gauge.

$$\text{Thus period} = 2\pi \cdot \sqrt{\frac{W}{g} \cdot \frac{L}{\pi r^2 Y}}$$

$$\text{and } Y = \frac{4\pi WL}{gt^2 r^2}$$

Results should be tabulated thus:

Original length of cord =
Original radius of cord =

Mass in pan.	Period of oscillation.	Young's modulus.

It may be mentioned that in the case of indiarubber no very concordant values of Y are obtainable.

39.

To determine the Modulus of Torsion for a Given Wire, and the Modulus of Rigidity for the Material of which it is made.—The modulus of rigidity of a substance is the ratio of the force per unit area of a plane, situated at unit distance from a parallel fixed plane, to the displacement of that plane in a direction parallel to itself, the force being supposed to act along the plane.

This displacement is termed the "shear," and the modulus is denoted here by N.

The modulus of torsion of a wire is that twisting moment which produces unit twist (1 radian) in unit length of it, one end being fixed, and the couple applied at the other. It may also be defined as the ratio of the twisting moment to the twist produced in unit length. It is denoted here by T.

To find the connection between N and T, let a circular wire of radius R and length l be taken for investigation.

The couple twisting this through unit angle is $\frac{T}{l}$.

Consider now a narrow ring-shaped element of the sectional area of radius x and width dx. The shearing force turning the end of this tube through unit angle at one end of the wire, its other end being fixed, is per unit area $\frac{Nx}{l}$, and the total force on the whole ring $\frac{2\pi x^2 dx N}{l}$. This acts at a distance x from the axis, so that its moment round it is $\frac{2\pi x^3 dx N}{l}$. Now, the sum of this quantity for all the rings in the wire's section is—

$$\frac{2\pi N}{l}\int_0^R x^3 dx = \frac{\pi N R^4}{2l}$$

$$\text{therefore } \frac{\pi N R^4}{2l} = \frac{T}{l}$$

$$\text{and } N = \frac{2T}{\pi R^4}$$

Two methods may be used in making an experimental determination.

(*a*) *The Statical Method.*—A wire is securely clamped above, and has a pulley attached rigidly to its lower end. Beneath this pulley is a disc with a graduated edge, or pointers moving over a scale. Two threads are wound round the pulley in opposite directions, and pass over other pulleys to pans. If the deflections are given by a disc, a telescope is focussed upon it. Loads sufficient to produce various twists should then be placed in the pans, an equal load in each, and

increased by steps, the deflection for each being noted. The same should be done, removing the weights by steps, afterwards. The mean of the readings given by the two pointers should be taken for each load in order that the error produced by any bodily shifting of the disc to one side or the other may be eliminated; the length of wire twisted is then measured, the diameter found by the micrometer gauge at several places, and the diameter of pulley found. The deflections, if read in degrees, must be converted into circular measure.

Results should be tabulated thus:

Length of wire =
Radius of wire =
Diameter of pulley =
Thickness of thread =

Load in each pan	Couple.	Deflection in degrees.		Deflection in radians.		Modulus of torsion.
		Increasing couple.	Diminishing couple.	Increasing couple	Diminishing couple.	

The couple = (load in one pan) × (diameter of pulley + thickness of thread).

The mean value of the modulus should be taken, and N for the material calculated.

Curves should be plotted for both the increasing and diminishing couple observations on the same axes, with couples as abscissæ and deflections as ordinates.

(*b*) *The Vibration Method.*—If the moment of inertia of the pulley and disc be I_1, and the threads be removed from the former, they will oscillate after deflection in a period equal to $2\pi\sqrt{\frac{I_1 l}{T}}$, for the movement is simple harmonic motion, and the acceleration of a rotating mass is the twisting moment at unit displacement divided by its moment of inertia. Let the observed period be t_1.

If I_1 be unknown, a body whose moment of inertia I_2 is known, conveniently a thin metal cylinder which may be fixed coaxially to the disc, may be added, I_2 for a cylinder being approximately mass × mean radius squared. The new period t_2 is then observed.

$$\text{Now, } t_1 = 2\pi\sqrt{\frac{I_1 l}{T}}$$

$$\text{and } t_2 = 2\pi\sqrt{\frac{(I_1 + I_2)l}{T}},$$

$$\text{therefore } t_2^2 - t_1^2 = \frac{4\pi^2 l I_2}{T}$$

$$\text{and } T = \frac{4\pi^2 l I_2}{t_2^2 - t_1^2}$$

The periods should be determined by the method explained in Art. 27.

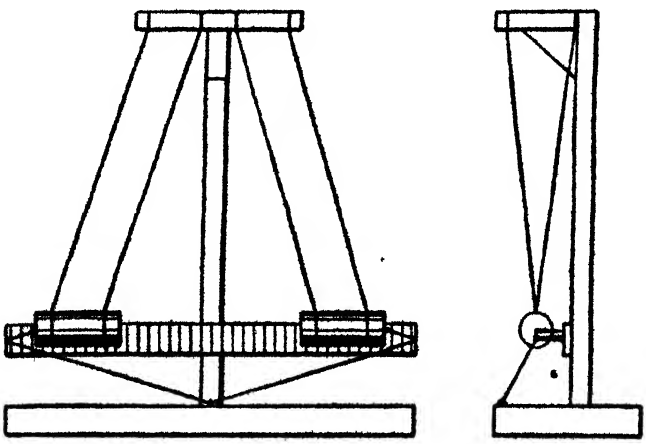
If T be known, the above method furnishes a means of determining the I of a rigidly attached body, about an axis in the same straight line as the wire.

If the volume elasticity be found by the method of Art. 36, and the rigidity modulus by calculation from the torsion results, then the ratio of the linear lateral contraction to the longitudinal elongation per unit length of a stretched wire may be found. For this ratio, known as Poisson's ratio, may be proved to be a constant and equal to $\frac{3K - 2N}{2(3K + N)}$, where K denotes the elasticity, and N the modulus of rigidity.

40.

To determine the Coefficient of Restitution of Substances in the Form of Bars by a Ballistic Method.—If two similar bodies of the same material impinge directly upon one another, the ratio of their momentum immediately after separation to that existing just before is termed the coefficient of restitution.

To determine this ratio ballistically, two similar bars are suspended by fine threads, so that their axes are in the same straight line and their faces just in contact when hanging freely. To each end of the bars are attached two threads as shown, so that while swinging the axis always lies horizontally in the same vertical plane. Parallel to the plane of motion are placed two scales, near to which move two light pointers attached to the outer ends of the bars, thus indicating the horizontal displacement of the bar when it is drawn away from the centre. The reading of one of these pointers when its bar hangs freely is noted, and the ends of a piece of thread are then fastened to



the outer ends of the bars. This thread is passed over two small pulleys as shown, so that upon its middle part being pulled and looped over a peg the bars are drawn apart. Their position is adjusted until each is displaced the same distance from its resting position, and this displacement is noted. The thread is now burned at its centre, releasing the bars, and the distance which the pointer originally noted rebounds back after collision is observed. The operations are repeated, the observations being made on the other bar. Under the conditions mentioned, the readings in each case should be identical; if they are not so, their mean should be taken. If the length of

the suspending threads is large compared with the horizontal displacements, the coefficient of restitution is given approximately by—

$$e = \sqrt{\frac{l}{L}}$$

where L is the original displacement, l that after rebound.

More accurately, cathetometer readings of the vertical distance h between the position of rest and at l, and H between the same and L, may be taken, the bars being drawn into the positions l and L by a thread.

$$\text{Then } e = \sqrt{\frac{h}{H}}$$

41.

To determine the Value of the Surface Tension of a Liquid.—Owing to the attraction between the molecules internally, the surface of separation between a liquid and any other substance is in a stretched condition, like a tight membrane.

The stretching force along unit length of any imaginary line on the surface is termed the "surface tension." The intensity of this varies according to the substances separated by the surface, but, unless otherwise stated, the surface between the liquid and air is understood.

Where three separating surfaces meet, as at the side of a vessel partly filled with a liquid, it is found that either the tension between the air and the solid, or that between the liquid and the solid, is greater than the sum of the other two tensions. Hence, in the former case, it pulls the liquid up the sides for some distance, as is the case with all liquids which wet the solid; or, in the latter case, it depresses it, as is the case with liquids which do not wet solids, *e.g.* mercury.

The surface tension of the first type of liquid may be determined by dipping a fine-bored glass tube into it. A column is drawn up to such a height that the downward pull of its weight

equals the upward pull of the surface tension round the inner circumference of the tube. Let the radius of the tube be R, the surface tension be T, the height of the column above the external liquid be H, and the density of the liquid be d. Then the weight of the column is $\pi R^2 H d$, and the upward pull of surface tension is $2\pi RT$.

$$\text{Therefore } \pi R^2 H d = 2\pi RT$$

$$T = \frac{RHd}{2}$$

As it is essential for an accurate measurement that the inside of the tube should be wetted for some little distance above the top of the column, it is well to plunge its lower end somewhat more deeply into the liquid at first, afterwards raising it nearer to the surface. The levels are measured by a cathetometer after the tube has been clamped in a truly vertical position. As the vessel containing the liquid does not usually permit the external level to be found with any accuracy, it is necessary to adjust a needle of known length in a clamp, so that it is held vertically near the tube and having its point just in contact with the external liquid surface. The vertical distance from the top of the column to that of the needle, plus the length of the latter, is then the total height of the column. The surface at the top of the column is a curved meniscus, and in reading its level, the cross-wire should be adjusted in the mean position between that of the centre and the sides. The internal diameter of the tube is found by cutting it across just above the top of the column, sticking it through a cork till the cut end just projects, and rubbing cork and tube on a smooth, wetted stone, after which it is placed under a reading microscope and measured.

In the case of the second type of liquid, the surface tension may be determined by partly filling a U-tube having one limb of fine bore and the other of greater diameter. The liquid is depressed in the former below the level of that in the wider limb. For let r be the radius of the narrow, R that of the wider tube, H the difference of level, d the density of the liquid, T the surface tension. Then the pressure per unit area

in the narrow tube is $\frac{2\pi r T}{\pi r^2} = \frac{2T}{r}$, and in the wider it is $\frac{2\pi R T}{\pi R^2} = \frac{2T}{R}$. The difference of these pressures equals that of the column supported.

$$\text{Hence } 2T\left(\frac{1}{r} - \frac{1}{R}\right) = Hd$$

$$\text{and } T = \frac{HdRr}{2(R - r)}$$

42.

To determine the Surface Tension of a Liquid by the Balance.—If a light metal plate be suspended horizontally from one arm of a balance and counterpoised, then adjusted until it lies upon the surface of a liquid and the balance pointer is at zero, the effect of adding weights to the pan is to raise the plate. Unless, however, the added weight is excessive, the plate is not dragged away from the liquid; but by reason of the surface tension round its perimeter draws up a short column of the liquid beneath itself. If the height of this column be H cms., the density of the liquid be d, the area of the plate be A, its perimeter be L, and the surface tension be T, it is evident that the total downward pull on the plate must now be—

$$HAd + LT$$

H may be determined by noting the ratio $\frac{\text{length of balance arm}}{\text{length of balance index}}$, and the position of the index. Denoting the ratio by k, the distance of the index from its zero in centimetres by l, and in scale divisions by n, the added weights by W, and the weight producing a deflection of one scale division by w—

$$H = kl$$

$$klAd + LT = (W - wn)$$

$$T = \frac{1}{L}(W - wn - klAd)$$

It is evident that T is more accurately measured as $\frac{L}{A}$ increases, so that the plate should be small. On this account a number of plates arranged parallel, and 5 mm. or more apart, in a vertical plane are advantageous. Their lower edges may then be immersed and the pull noted. If W be the weights added to keep the index at zero, and there be N plates, each L′ cms. wide at the surface of the liquid—

$$2NL'T = W$$

$$\text{and } T = \frac{W}{2NL'}$$

The lower edges should not dip deeply into the liquid, as corrections for the buoyancy of the liquid would then become necessary.

43.

To determine the Surface Tension of a Liquid by the Method of Drops.—If a liquid be allowed to flow slowly through a small tube clamped vertically, it falls intermittently in drops from the lower end. The size of the drop is determined by the fact of its detaching itself when its weight equals the total pull of the surface tension round the circumference of the tube. Thus, if W be the weight of one drop, R the radius of the lower end of the tube, T the surface tension—

$$W = 2\pi RT$$

The lower end of the tube, which may conveniently be of several millimetres diameter, should be ground off to a sharp edge perpendicular to the length. A short piece of thermometer tubing connected by rubber tube to the dropping tube and to a funnel held in a retort stand prevents an unduly rapid flow. These being in position, the liquid is poured into the funnel, and, if necessary, the length of thermometer tubing adjusted until the dropping is sufficiently slow. A beaker of known

weight is then placed to receive the drops, and a large counted number allowed to fall into it, after which it is again weighed. The increase of weight divided by the total number of drops gives W. The radius of the tube is measured under a reading microscope, and T calculated from the relation—

$$T = \frac{W}{2\pi R}$$

This method is not very reliable, owing to the difficulty of estimating what proportion of the water column's hydrostatic pressure is supported by the drop.

44.

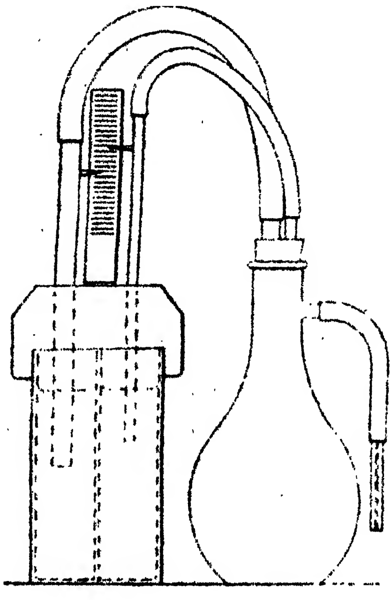
To determine the Surface Tension of a Liquid by Jager's Method.—A tube of known diameter is clamped vertically with its lower end, which is ground off sharp and square, immersed in the liquid, and the minimum air-pressure in the tube which liberates bubbles from its lower end is observed. Since the pressure inside a bubble is equal to $\frac{2T}{r}$, where T is the surface tension, and r the radius of the bubble, it increases until the bubble is hemispherical. Beyond this point, however, the pressure due to surface tension is reduced as the volume increases, for r is increased. Thus the conditions are unstable, and the bubble is detached.

That air-pressure, P, which is just sufficient to produce bubbles consequently satisfies the equation—

$$P = \frac{2T}{r} + Hd$$

where d denotes the density of the liquid, and H the depth of the end of the tube below the surface. This value of H is only approximate, but it may be taken as sufficiently accurate when the diameter of the tube is small compared with the depth to which it is immersed. In practice, comparative observations are more satisfactorily made. Two tubes of unequal diameters, but connected to the same air supply, are immersed together in one of the two liquids of which the surface tensions are to be compared, and their depths adjusted until bubbles are liberated

at the same rate from each. As the disturbance of the liquid when a bubble from one tube is liberated is apt to distort and liberate the one on the other tube, a diaphragm must be placed in the liquid between the tubes. Let H_1, d_1, and T_1 be the difference of depth between the tubes, the density of the liquid, and its surface tension respectively.



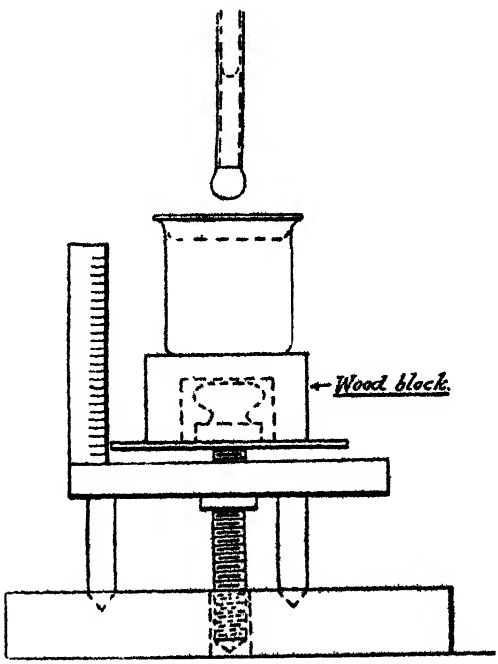
The tubes are now immersed in the second liquid and the same observation made. Let H_2, d_2, and T_2 have the same meanings as before. Then—

$$\frac{T_1}{T_2} = \frac{H_1 d_1}{H_2 d_2}$$

The air-pressure may very conveniently be produced by blowing through a piece of thermometer tubing into a flask, which acts as a regulator, and is connected to the bubble tubes.

45.

To determine the Surface Tension of a Liquid by Sentis's Method.—The advantage of this over the ordinary capillary tube method is that the measurement of the internal diameter, in which a large percentage error is probable, is avoided. A fine-bored tube is filled with the liquid, and a



part of the latter allowed to flow out, forming a suspended drop at the end of the tube, which is held vertically in a stand. The maximum diameter of the drop is measured by a microscope with a micrometer eye-piece, and the position of the meniscus at the surface of the liquid inside the tube is

marked. A beaker of the liquid is now placed below the drop and raised until it just meets it, emptying the tube. The position of the beaker is noted, and it is then further raised until the end of the tube is immersed sufficiently far to bring the meniscus of the drawn-up liquid to its original position. The position of the beaker is again noted. These two observations of the height of the beaker may very conveniently be made by boring three small pits and a central hole in a block of wood to receive the legs of a spherometer, supporting a small beaker on the disc of the latter, and adjusting so that with the central leg screwed well down the surface of the liquid in the beaker is just below the drop.

The vertical distance between the two positions of the beaker is equal to the height of the column of liquid which would be drawn up by a tube equal in internal diameter to the measured diameter of the drop. But denoting these by H and D—

$$T = \frac{HD}{4} \times \text{density of the liquid}$$

46.

To determine the Surface Tension of a Liquid by Quincke's Method.—In the case of liquids which do not wet the surface with which they are in contact, a large flat drop is obtained upon a horizontal plane. By means of a cathetometer, the vertical distance between the upper flat surface of the drop and that point on its curved edge at which the tangent is vertical is measured. If H be this height, it may be proved that, very approximately—

$h_1 - h_2 = H.$

$$T = \frac{H^2 d}{2}$$

T being the surface tension, and d the density of the liquid.

Liquids which do wet solids necessitate the employment of a flat plate fixed horizontally on legs inside a glass vessel, one side of which should be plane. The liquid is poured in to a sufficient depth to cover the plate, and a large air-bubble is blown underneath the latter by means of a bent tube. The same measurement is made as in the previous case, but is now taken upwards from the lower surface of the bubble.

The degree of approximation to accuracy depends upon the extent to which the curvature of the drop or bubble in a horizontal plane is negligible in comparison with that in a vertical plane; hence it is necessary that the horizontal diameter should be large.

47.

To determine the Surface Tension between Two Liquids which do not mingle.—The most simple method is a modification of that by drops (Art. 43). A quantity of the less dense liquid is poured into a vessel, and both are weighed together. A tube is clipped vertically with its lower end immersed in the vessel of liquid, and the denser liquid is allowed to pass slowly down it, falling in drops from its end. After some large counted number of drops have fallen, the vessel is removed and again weighed. The increase in weight being denoted by W, and the number of drops by N, the weight in air of one drop is $\frac{W}{N}$. If d_1 be the density of the lighter, d_2 of the heavier, liquid, the weight of the drop in the lighter liquid is $\frac{W(d_2 - d_1)}{Nd_2}$, and this was supported by the surface tension round the circumference. Hence R being the radius of the tube, and T the surface tension between the liquids—

$$2\pi RT = \frac{W(d_2 - d_1)}{Nd_2}$$

$$T = \frac{W(d_2 - d_1)}{2\pi RNd_2}$$

The method of Art. 46 may also be applied, a large drop of the denser liquid being produced on a horizontal plane immersed in the less dense liquid. In this case the expression for the surface tension becomes—

$$T = \frac{H^2(d_2 - d_1)}{2}$$

48.

To determine the Viscosity of a Liquid by the Method of Poiseuille.—The viscosity of a liquid is the force required per unit area to give unit velocity to a surface of the liquid at unit distance from a fixed parallel surface.

The quality of liquids commonly termed "thinness" or "thickness" depends upon the viscosity.

It may be proved that if a liquid be forced slowly through a fine-bored tube of radius R, and length L, by some pressure P, the volume V driven through in t seconds is equal to—

$$\frac{\pi PR^4t}{8ML}$$

where M is the value of the viscosity. Hence—

$$M = \frac{\pi PR^4t}{8LV}$$

The length of the tube to be employed, conveniently thermometer tubing, is measured, and its radius found by the method of filling with mercury (Art. 10). One end is connected by rubber tubing to a vessel of the liquid, and some allowed to flow through. A vessel containing some of the

liquid into which the lower end of the tube may dip is then weighed, and the lower end immersed in it. At the instant of doing this the time and the difference of level between the supplying and receiving liquid surfaces are noted. After twenty or more cubic centimetres have passed through, the tube is withdrawn from the vessel, the same observations being made as before. By weighing the receiving vessel again, and dividing by the density of the liquid, the volume V is obtained. The mean of the initial and final differences in level multiplied by the density of the liquid is the average pressure P. Hence M may be calculated. As the temperature coefficient of the viscosity is large, the temperature of the determination should be stated.

The viscosities of two liquids, where the same apparatus and the same mean difference of levels are employed, have the ratio—

$$\frac{M_1}{M_2} = \frac{V_2 d_1}{V_1 d_2}$$

i.e. they are inversely as the volumes of liquid passing through in equal times, and directly as their densities.

The tube should be so fine-bored and long that the liquids pass through but slowly under the pressure applied.

49.

To compare the Viscosities of Liquids by the Vibrating Disc Method.—If a disc be rigidly fastened to a wire, so as to oscillate slowly under its moment of torsion when displaced through an angle, its oscillations become damped, and ultimately it comes to rest. The energy has been expended in imparting motion to the surrounding fluid, and producing heat by molecular friction in the wire. When the surrounding fluid is a liquid the latter term is negligible, and the work done in imparting equal velocities to different liquids is proportional to their viscosities.

Now, the energy lost by the disc per unit time through

viscosity is proportional to its velocity, and the rate at which the amplitude is diminishing is proportional to the product of itself and the viscosity. Let A_m denote any amplitude, k the viscosity of the liquid, and c a constant depending on the form and size of the disc.

$$\text{Then } \frac{dA_m}{dt} \propto -cA_m k$$

$$\text{therefore } A_m \propto \epsilon^{-ckt_1}$$

$$\text{Similarly, } \frac{dA_n}{dt} \propto -cA_n k$$

$$\text{therefore } A_n \propto \epsilon^{-ckt_2}$$

Now let there be N vibrations between the mth and nth Then $t_2 - t_1 = N \times T$, where T is the vibration period.

From the above equations—

$$\frac{A_m}{A_n} = \frac{\epsilon^{-ckt_1}}{\epsilon^{-ckt_2}} = \epsilon^{ckNT}$$

and therefore—

$$k = \frac{\log_e R}{cTN}$$

where R is the ratio of the extreme swing of one oscillation to that of the Nth succeeding one, the first noted one not being included.

The weight of the disc should be considerable that the oscillations may not die out too quickly, or be too rapid; an index is attached to its axis, and moves over a scale.

Observations should be made with several mean amplitudes and results stated thus:

First liquid. T =

First noted amplitude.	Nth amplitude.	Ratio R.	N.	$\frac{\log_e R}{TN}$

Second liquid. T =

First noted amplitude.	Nth amplitude.	Ratio R	N.	$\frac{\log_e R}{TN}$

The mean values of the entries in the last column should be found for each liquid, and their ratio calculated.

50.

To determine the Relation between the Pressure and Volume of Air at Constant Temperature, and the Isothermal Elasticity.—For this purpose a U-tube with one limb shorter and closed, the other longer and open, is employed. The tube is of uniform bore, so that lengths and volumes of it have the same ratios.

Clean mercury is poured into the open arm enclosing a volume of air in the shorter. The tubes being quite vertical, the length of the air-column and the heights of the mercury in each arm are noted. More mercury is then poured in, shortening the air-column by a centimetre or less, and the observations made as previously.

This operation is repeated until the longer arm is nearly full. The height of the barometric column is then observed, and the pressures on the enclosed air found by adding it to the differences of level in the arms, *i.e.* in centimetres or inches of mercury.

Results should be tabulated thus:

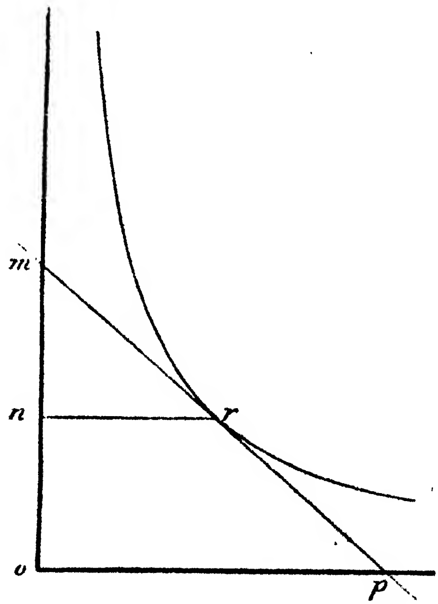
Height of level in closed arm.	Height of level in open arm.	Difference of levels.	Total pressure.	Length of air column.	Product of pressure and length.

A curve should be plotted with volumes ($\propto$ lengths) as abscissæ and pressures as ordinates.

The law of relation between pressure and volume, which should be apparent from the last column, is called "Boyle's law."

The elasticity of a substance, gaseous or otherwise, is the ratio of a change in pressure to the change in volume per unit volume, produced by it. When the temperature remains constant, the elasticity is said to be "isothermal."

To find the elasticity from the curve, a tangent should be drawn to it at any point, intersecting the axes. The slope of



this line shows the rate at which the pressure is changing with change of volume. Let it cut the axis of ordinates in m, that of abscissæ in p. Through the point at which the tangent is drawn, let nr be drawn parallel to op.

$$\text{Then the ratio } \frac{\text{change of pressure}}{\text{change of volume}} = \frac{om}{op} = \frac{nm}{nr}$$

and the elasticity equals this multiplied by the volume. But the latter is represented by nr, therefore the elasticity $= nm$; and from the properties of the curve, $nm = on =$ the pressure.

Isothermal elasticity = the pressure on the gas

Boyle's law may also be proved by means of a straight glass tube of about 3 mm. bore, closed at one end, and having a thread of mercury several centimetres long introduced some little way in from the open end. The method may be left as an exercise to be devised.

51.

To determine the Volume of a Body by Means of the Volumenometer.—A volumenometer consists of a vessel, with an opening which when closed is air-tight, connected to a pair of vertical tubes connected at their lower ends by rubber tubing. The volume per unit length of the tube to the top of which the vessel is connected must be known, and the vertical position of the other must be adjustable, so that, by raising or lowering it when partially filled with mercury, the pressure on the air in the vessel may be varied.

This description applies to a simple form of apparatus easily set up. In more elaborate forms the tubes are fixed and provided with taps at the bottom, from one of which mercury can be run out when adjusting levels.

As always, the mercury to be used must be clean, and the tubes quite dry; while, in exact work, the air in the vessel should be dried. The dryness of the air in the vessel may be ensured by blowing air into it for some time through drying tubes of sulphuric acid on pumice before closing.

The vessel is sometimes in the form of a cup, and is closed by laying a flat piece of glass over the mouth, the air-tight joint being made by tallow previously smeared on the edge. A flask with a side tube, closed by a rubber cork, is simpler, and yields fair results.

Whatever the form of apparatus, it should be placed in a wooden tray to avoid probable loss of mercury.

In use, the tubes are firstly adjusted to be vertical by a plumb-line. If the air has been dried, the tubes should previously have been filled with mercury up to a noted position near the vessel. Otherwise, the vessel is now opened and the same done. The cork or cover is then replaced carefully to make an air-tight joint, and the height of the movable tube adjusted until the levels are exactly at the same height in both tubes. This height should be noted. The enclosed air is now under atmospheric pressure. The open limb is then lowered considerably, reducing the pressure on the air and allowing its volume to increase by an amount which may be read. Let this increase be v, and the difference in height of the columns be h. Let also the observed height of the barometer be H, and the original volume of air be V.

$$\text{Then } VH = (V + v)(H - h)$$

$$\text{whence } V = \frac{H - h}{h} v$$

The body of which the volume is required is then placed inside the vessel, and the whole operation repeated, giving some new value V′. Consequently the required volume is V − V′.

The volumenometer may thus be used to determine densities of porous and loose substances; it also serves to verify Boyle's law.

52.

To determine Heights by the Barometer.—This determination is only conveniently made by means of an aneroid, but some corrections, which, for accuracy, should be made in reading the height of the mercurial form, may be here alluded to.

Assuming that the column is adjusted perfectly vertical, the chief corrections are—

(*a*) To correct for expansion of the mercury.

(*b*) For expansion of the scale.

These are embodied in the expression—

$$H' = H - (c_m - c_s)Ht$$

where H′ is the corrected height, H that observed, c_m, c_s the cubical expansion of mercury and the linear expansion of the scale respectively, t the temperature. The last term $(c_m - c_s)Ht$ may usually be put $(c_m - c_s)760t$.

(*c*) For the vapour-pressure of the mercury which may be found from tables, and which must be added to H′.

(*d*) For altitude above sea-level.

Other minor corrections are those for capillary depression, and variation in the value of g.

To determine heights, let a vertical column of air of unit area be supposed divided into n layers, each so short that its density may be considered uniform. If the column be of height h, each layer will be $\frac{h}{n}$ thick.

Let d and p be the density and pressure of air at the earth's surface; $d_1 p_1$, $d_2 p_2$, etc., those of the succeeding layers.

$$\text{Then } p - p_1 = d\frac{h}{n}$$

$$\text{and by Boyle's law } \frac{p}{d} = \frac{p_1}{d_1} = \frac{p_2}{d_2}, \text{ etc.,} = \text{R suppose}$$

$$\text{so that } R(d - d_1) = d\frac{h}{n}, \text{ and } d_1 = \left(1 - \frac{h}{Rn}\right)d$$

Similarly—

$$d_2 = \left(1 - \frac{h}{Rn}\right)d_1, \quad d_3 = \left(1 - \frac{h}{Rn}\right)d_2, \quad d_n = \left(1 - \frac{h}{Rn}\right)d_{n-1}$$

Thus the density decreases in a geometrical progression as the heights increase in an arithmetical one.

$$\text{Therefore } \frac{d_n}{d} = \left(1 - \frac{h}{Rn}\right)^n$$

Let H be the height of the barometer at a place, and H′

that at a higher position. These heights are proportional to the density of the air at each, so that—

$$\frac{H'}{H} = \left(1 - \frac{h}{Rn}\right)^n$$

and when n is infinite—

$$\frac{H'}{H} = \epsilon^{-\frac{1}{R}h}$$

$$\frac{H}{H'} = \epsilon^{\frac{1}{R}h}$$

$$h = R(\log_\epsilon H - \log_\epsilon H')$$

h being the height between the two positions.

When h is known to be such that the density of air does not vary much in it, the lengths H and H′ may be used instead of their logarithms.

This latter form is sufficiently accurate for determination of the height between the basement and upper floor of a building.

53.

To calibrate a Small Aneroid Barometer.—For indications below the atmospheric pressure the aneroid should be placed under the bell-jar of an air-pump in such a position that its scale is clearly visible.

A mercury gauge is then connected to the pump so as to be in communication with the air inside the bell-jar.

The latter being well sealed down with a tallow joint, a partial vacuum is produced. As some leakage may occur, this vacuum will probably fall. In this case the instant the index of the aneroid reaches some noted division, the connection between the bell-jar and the gauge should be cut off, either by a tap or pinchcock. The vertical distance between the mercury levels in the gauge should then be read by cathetometers, or from verniers sliding along scales permanently fixed to the tubes.

The gauge is then again put in connection, and a greater vacuum produced, the same operations being repeated.

The height of the mercury barometer should be read at intervals, and the necessary corrections applied (Art. 52). If its height perceptibly varies, it must be observed for each reading of the gauge.

The pressure inside the jar is equal to that of the mercury barometer, minus that shown by the gauge.

Results should be tabulated thus:

Aneroid reading.	Gauge reading.	Height of barometer.	Pressure on aneroid.	Correction.

A curve should be drawn having aneroid readings as abscissæ and the corrections to a large scale as ordinates.

By placing the aneroid inside a glass vessel into which air may be compressed, corrections may be found for pressures greater than that of the atmosphere.

54.

To construct a Barometer by inserting a Bead of Mercury into a Glass Tube of which One End is closed, and to make a Scale of Temperature Corrections.—The tube should have a uniform bore of a few millimetres, and a length of not less than half a metre. By heating it, and then dipping into mercury, a thread of several centimetres' length is drawn up so that it occupies a position about 10 cms. from the open end when the tube is horizontal. Let the length of that part of the horizontal tube occupied by the enclosed air be l, the temperature after cooling to that of the atmosphere be t°, and the barometric height be H. Then, at 0° C., the length l would become—

$$l' = l\frac{273}{273+t}$$

and under 760 mm. pressure the length of the air-column l' would become—

$$L_{760} = l\frac{273}{273+t}\cdot\frac{H}{760} = l'\frac{H}{760}$$

A table of lengths should now be calculated for every centimetre difference of pressure, say, between 660 and 860 mm. pressures—

$$L_{660} = l'\frac{H}{660}$$

$$L_{650} = l'\frac{H}{650}$$

$$L_{640} = l'\frac{H}{640}$$

$$\vdots = \vdots$$

$$L_{860} = l'\frac{H}{860}$$

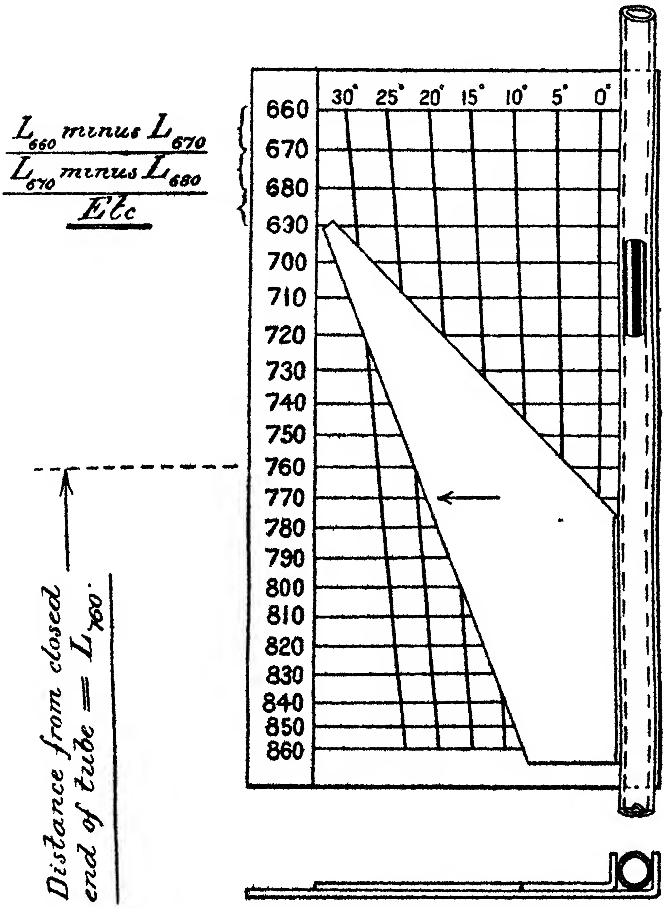
and parallel lines drawn on a piece of paper or card to form a scale, their distances apart being as shown in the diagram. Let the temperature corrections be required for every five degrees between 0° and 30° C. Then the deductions to be made from an observed length L_{860} are for various temperatures as follows:—

At 5° C. . . . $\frac{5}{278}L_{860}$

At 10° C. . . . $\frac{10}{283}L_{860}$, etc.

These are calculated for L_{660} and L_{860}, and marked off on the pressure lines from a line of zero, after which points denoting the same temperature are joined by straight lines. These lines are numbered with the temperature to which they refer, and from any pressure line they cut off the lengths to be deducted. By cutting a piece of card so that two of its sides are straight and at an angle of 135° with each other, an easy means of applying the correction is obtained; for one straight side being adjusted to the tube, and moved along it until the other passes through the intersection of the observed pressure and temperature lines, the correct pressure is given where this side cuts the line of zero

temperature. Obviously the scale of pressures must be constructed for a greater range than that to which the corrections



can be applied, the full range given being only available at zero.

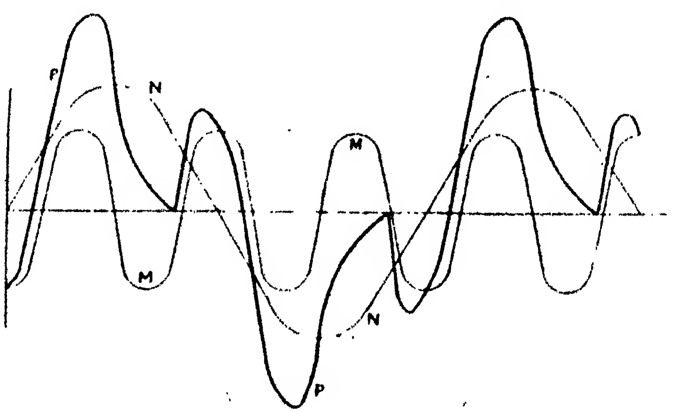
The tube may be graduated to give indications when in a vertical position, if in calculating the lengths it be noted that

the internal air-pressure is greater or less than the atmospheric by the length of the bead—greater when the open end is upward, and less when it is downward.

The indications of an instrument of this type are not very exact owing to the sticking of the mercury—a defect which is greatly increased if the tube be at all damp. Error is also introduced by water-vapour unless the air inside is perfectly dry.

55.

To plot the Curve of Resultant Values of a Quantity which is the Sum of Others, the Others varying periodically in any Manner, and having any Phase Difference.—Let M and N be two curves in which the ordinates denote the values



of the components, those below the axis of abscissæ being negative. The resultant of these is found by drawing ordinates equal to the algebraical sum of the two, and the curve P so obtained is that of total effect.

56.

To determine graphically the Harmonic Components of any Periodic Function.—It is proved by Fourier's theorem

that any given periodic curve, whether itself harmonic or otherwise, may be analyzed into a number of harmonic components. These components in general differ in period, amplitude, and phase.

Writing the equation of the function or curve as—

$$f(x) = A_0 + A_1 \sin \frac{2\pi}{X}x + B_1 \cos \frac{2\pi}{X}x + A_2 \sin \frac{4\pi}{X}x$$
$$+ B_2 \cos \frac{4\pi}{X}x + \text{etc.}$$

where X is the value of x for one entire period, the values of coefficients may be found in the following manner, as described by Professor J. Perry (*Electrician*, June, 1895).

Supposing N corresponding values of $f(x)$ to be known, $A_0 = \{\text{the sum of the N values of } f(x)\} \div N$.

A line, PQ, is drawn, and a circle of $\frac{N}{2\pi}$ inches' radius described about it. The circumference of this is divided into N parts, and from each a perpendicular is drawn cutting PQ. The divisions on the circle are numbered 0, 1, 2, etc., N, and the corresponding points on PQ similarly marked.

A straight line, PS, is drawn in any convenient position perpendicular to PQ, and points 0, 1, 2, etc., N, are marked upon it at distances from P, proportional to the given values of $f(x)$ when $x = 0, 1, 2$, etc., N. By projecting horizontally and vertically from similarly numbered points in PQ and PS, and drawing a line through their points of intersection, the curve A_1 is obtained. The area enclosed by this, when divided by $\frac{N}{2}$, gives the coefficient A_1 in the equation.

By projecting from 0 on PS to intersect the projection from $\frac{N}{4}$ on PQ, and obtaining the whole series of intersections as below—

Projecting point on PS	0	1	2	etc.
" " PQ	$\frac{N}{4}$	$\frac{N}{4}+1$	$\frac{N}{4}+2$	

a curve is drawn of which the area divided by $\frac{N}{2}$ is equal to the coefficient B_1. Projecting as follows:—

Projecting point on PS	0	1	2	3	etc.
" " PQ	0	2	4	6	

and dividing the area enclosed by N, the coefficient A_2 is found.

B_2 is given by the area of the curve through the intersections—

Projecting point on PS	0	1	2	3	etc.
" " PQ	$\frac{N}{4}$	$\frac{N}{4}+2$	$\frac{N}{4}+4$	$\frac{N}{4}+6$	

divided by N.

A further term, A_3, is given by—

Projecting point on PS	0	1	2	3	etc.
" " PQ	0	3	6	9	

and dividing the area enclosed by the curve by $\frac{3}{2}$N.

B_3 is given by—

Projecting point on PS	0	1	2	3	etc.
" " PQ	$\frac{N}{4}$	$\frac{N}{4}+3$	$\frac{N}{4}+6$	$\frac{N}{4}+9$	

the area of the curve being divided by $\frac{3}{2}$ N.

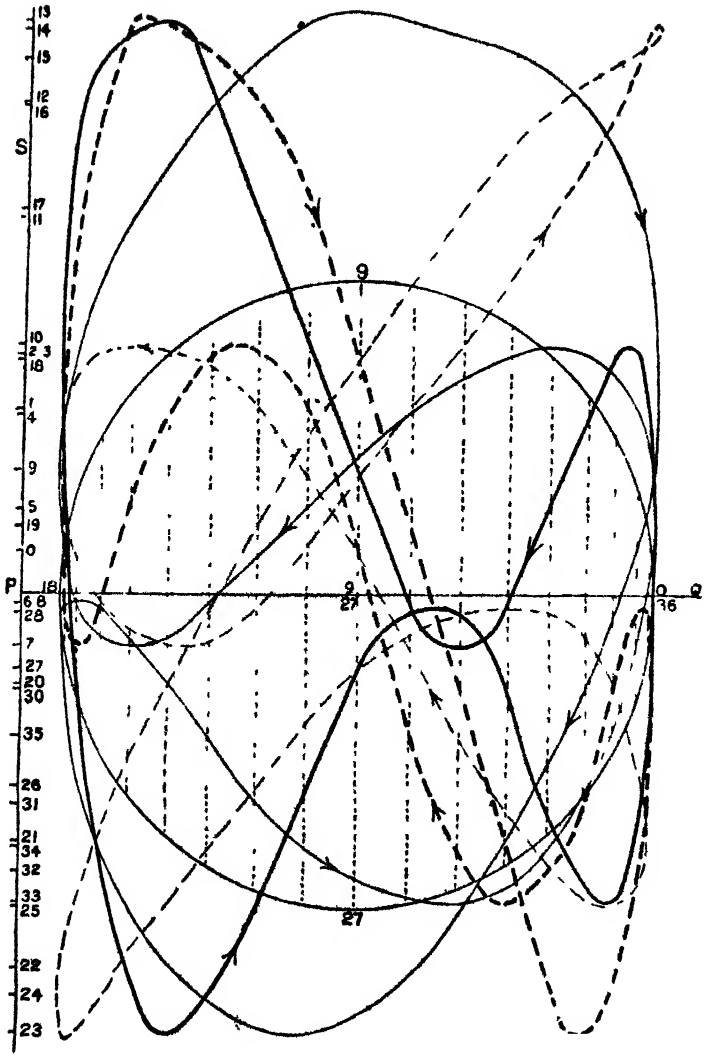
Generally, the points of intersection for A_n are—

Projecting point on PS	0	1	2	3	etc.
" " PQ	0	n	$2n$	$3n$	

and A_n equals the area of the curve divided by $\frac{n}{2}$N. For B_n—

Projecting point on PS	0	1	2	3	etc.
" " PQ	$\frac{N}{4}+n$	$\frac{N}{4}+2n$	$\frac{N}{4}+3n$	$\frac{N}{4}+4n$	

and the area of the enclosed curve is divided by $\frac{n}{2}$N.


S
P
Q
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2 3
18
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5
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7
27
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32
33
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22
24
23
9
27
36

FIG. 1.

The function may be expressed in the form—

$$f(x) = A_0 + D_1 \sin\left(\frac{2\pi}{X}x + \delta_1\right) + D_2 \sin\left(\frac{4\pi}{X}x + \delta_2\right)$$
$$+ D_3 \sin\left(\frac{6\pi}{X}x + \delta_3\right) + \text{etc.}$$

by making $D_1 = \sqrt{A_1^2 + B_1^2}$

$$D_2 = \sqrt{A_2^2 + B_2^2}$$
etc.

and $\delta_1 = \tan^{-1}\dfrac{B_1}{A_1}$

$$\delta_2 = \tan^{-1}\frac{B_2}{A_2}$$
etc.

It is to be noted that the areas are to be reckoned positive or negative according as the curve advances in a counter-clockwise or clockwise direction, and that consequently, if measured by a planimeter (Art. 6), the sign of the instrument's reading must be reversed.

In Fig. 1 thirty-six values of a recurring cycle are set out; the thick-line curve is that giving A_1; the thick-dotted curve, that giving B_1. The fine-line curves similarly give A_2 and B_2.

The following method of analyzing a curve, due to Wedmore, is very simple and convenient. The areas enclosed by the positive and negative portions are measured, and their difference, divided by the length of axis corresponding to an entire period, is set off above or below the original axis, according as it is positive or negative. The new axis of abscissæ is now drawn through this point. If the curve be divided into two equal parts along this axis, and these be superposed, the resulting curve contains only the components of 2, 4, 6, etc., times the frequency of the original, and its ordinates are twice their natural length. Restoring these to their correct dimensions by plotting to half the original scale, and taking the fourth as the highest harmonic that need be considered, this last is obtained, again to double scale, by superposing the

halves of the last curve. Plotting these to half scale, and deducting them from that curve, the component of twice the original frequency is yielded to its true scale.

By dividing the original curve into three equal parts along the axis, and superposing these, a curve is obtained which contains, to three times the true scale of ordinates, all components of 3, 5, 7, etc., times the original frequency. As all above the first of these are being neglected, the resulting curve set out to one-third the original scale is the harmonic of three times the original frequency.

To obtain the fundamental variation of the same frequency as the original curve, the three curves already obtained are set out, and the algebraic sums of their ordinates deducted from those of the original.

Thus, the variations of a certain quantity which recur periodically at equal intervals of time are plotted on a time base by the thick-line curve M of Fig. 2. The axial length AB, corresponding to the whole period, includes 36 intervals, at each of which the quantity represented by the ordinate was noted.

The difference of positive and negative areas is found to be -36; the true axis will, therefore, lie at distance 1 below the original axis.

The curve M is now divided and superposed as in Fig. 3, the resulting curve N being plotted to half scale.

In Fig. 4 the curve N is again subdivided into two and superposed, giving P, which, plotted to half scale, is the component of four times the original frequency.

P being deducted from N (Fig. 5), the component R of twice the original frequency remains.

In Fig. 6 the original curve, M, is trisected and superposed, giving S, which is the component of three times the original frequency, and whose ordinates are erected to one-third the scale.

Curves P, N, and S are now described on the new axis of M, and the sum of their ordinates deducted from those of M. The resulting curve is the component U of the original frequency.

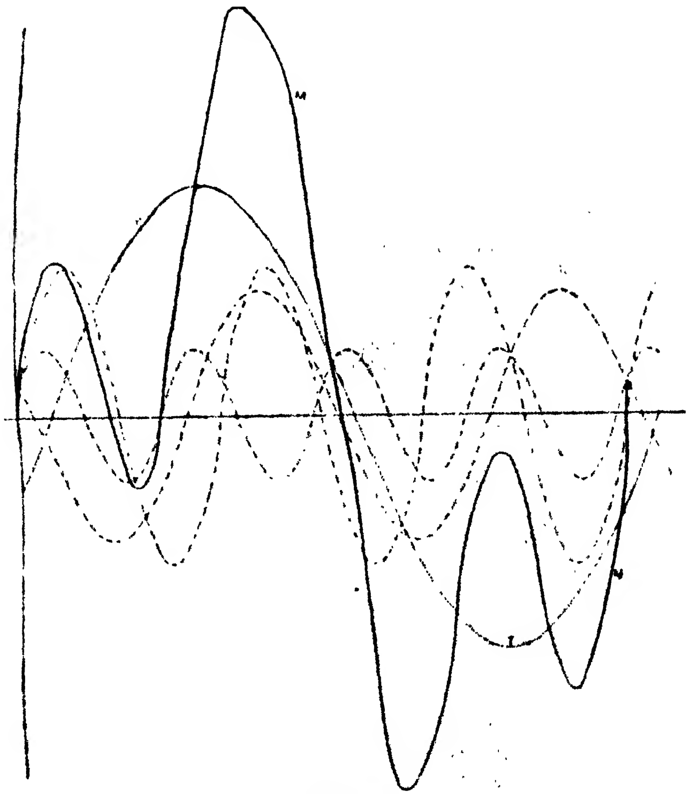


FIG. 2.

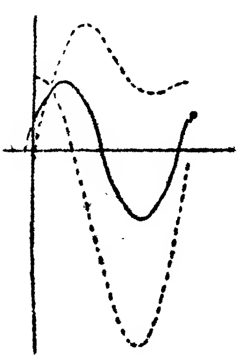


FIG. 4.

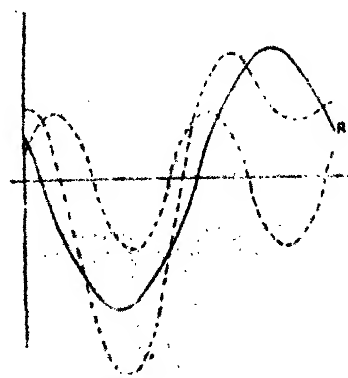


FIG. 5.

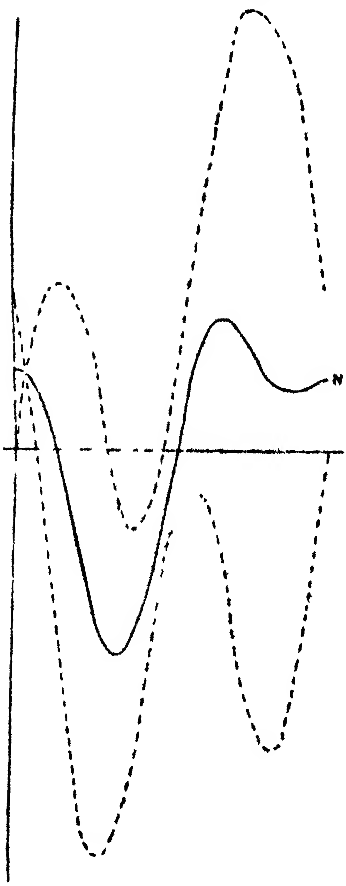


FIG 3.

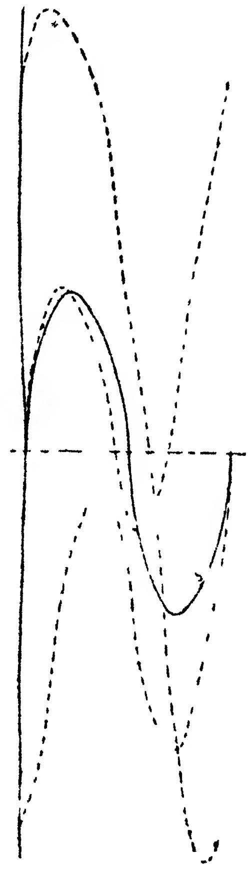


FIG. 6.

The phase of the curves is obvious from the setting out, and the expression for the observed quantity is—

$$1 + 7 \sin\left(\frac{2\pi}{T}t - \frac{\pi}{9}\right) + 3{\cdot}8 \sin\left(\frac{4\pi}{T}t + \frac{8\pi}{9}\right) + 4{\cdot}5 \sin\left(\frac{6\pi}{T}t\right)$$
$$+ 2 \sin\left(\frac{8\pi}{T}t + \frac{\pi}{8}\right)$$

PART II

EXPERIMENTAL DETERMINATIONS IN HEAT

57.

To determine the Boiling-point of a Liquid under Atmospheric Pressure.—The temperature of a boiling liquid depends on several circumstances, but that of its vapour is a constant for any one particular pressure. Unless otherwise specified, the boiling-point refers to a pressure of 760 mm. of mercury.

The liquid should be heated in a test-tube which is closed by a cork, through which passes a thermometer and a glass tube to carry off the vapour. The thermometer-bulb must be above the level of the liquid.

If the vapour be inflammable, it should be led by tubing well away from the heating flame; also, if the boiling-point be below 100°, it is as well to heat the liquid by placing a vessel of water so as to immerse the lower end of the test-tube. This vessel of water may then be heated, and all danger from possible fracture of the tube avoided.

Such apparatus constitutes a simple form of **hypsometer**, so called because by observing the variations of temperature at which boiling occurs at different heights above sea-level, the corresponding altitudes may be inferred.

58.

To determine the Temperature of the Liquid when Boiling occurs in Salt Solutions of Various Concentrations.—In works on physical chemistry it is shown that the molecular weight of a compound may be deduced from the observed boiling temperature of dilute solutions. In the present investigation it is not proposed to work with this object, but to discover the general effect produced. A sufficient quantity of distilled water to fill a flask to the base of its neck, preferably an even number of hundreds of grams, is weighed or measured in, and a mark made on, the flask at the level of the surface.

The temperature of the boiling liquid is then found by cautiously putting a thermometer in the flask when boiling occurs. A weight of salt equal to one per cent. of the water's weight is then added, and the temperature when boiling again noted.

The same weights of salt are repeatedly added, and the temperature of the boiling solution for each strength noted, until no further salt dissolves.

A few drops of water should be added from time to time to maintain the quantity in the flask unchanged.

It should be noted that the temperature of the vapour remains throughout at 100°.

Results should be tabulated thus:

Percentage of salt.	Temperature of boiling liquid.

A curve should be plotted with percentages of salt as abscissæ and elevations of the boiling temperature as ordinates.

59.

To determine the Fusing-point of a Solid which liquefies below 100°.—Some of the solid having been placed in a test-tube, is fused by immersing the end of the tube in water, which is then heated to a few degrees above the melting temperature.

A thermometer-bulb is then dipped into the tube, and, having risen to the same temperature, is withdrawn wetted with the fused substance.

It is closely watched, and the temperature at which the film on its bulb sets is noted.

The bulb is then dipped into water, which is slowly heated, and the temperature at which the film on it fuses and becomes transparent is noted.

These operations should be repeated several times, and the mean temperature taken as the fusing-point.

If the solid be soluble in water, a heated body may be slowly brought near the bulb, and the temperature at which the film fuses noted; while, if the fusing-point be above 100°, a sand-bath may be used in place of the water-bath, and the first method employed.

Another method is to draw out a fine glass tube into which a short column of the fused substance is sucked up. After solidification the small tube is tied, or secured by rubber bands, to a thermometer in such a way that the substance is close to the thermometer-bulb. The combination is then immersed in water, and slowly heated, until the substance fuses and becomes transparent. As soon as this is observed to begin, the temperature is noted, and the source of heat removed. Cooling now commences, and the temperature at which the substance begins to solidify and become opaque is noted. The mean of these two temperatures is taken as that of fusion.

The fusing-point may also be found by heating a quantity of the substance in a test-tube to a temperature above that of fusion, and reading the temperature at every minute during its cooling, by means of a thermometer dipping into the substance.

To protect the tube from external influence, it is as well to let it cool while supported inside a larger tube or jar, though this is not essential. At one period of cooling, viz. that during which the substance solidifies, the temperature falls but slowly; observations should be continued well past this stage. Finally, a curve should be set out with times as abscissæ and temperatures as ordinates. The mean temperature of the nearly horizontal portion may be then taken as the fusing-point.

60.

To determine the Coefficient of Linear Expansion of a Substance.—The coefficient of linear expansion of a substance is the fraction of its length at 0° C. that a rod of that substance changes in length per degree Centigrade rise of temperature; in other words, it is the fraction of a centimetre that a rod 1 cm. long at 0° C. changes in length for 1° rise of temperature. The mean coefficient between 0° and 100° may be found by heating a rod from 0° to 100°, and dividing its alteration in length by 100 and by its length at 0°. An approximate result may be obtained by measuring the length at t° (the temperature of the air) and at 100°, dividing then the change in length by $(100 - t)$ (length at t°).

A satisfactory method is to support the rod, the length of which may be from half a metre upwards, inside a tube of larger diameter. The ends of the tube should be closed by thin caps, conveniently of sheet rubber, through which pass the rod and small tubes whereby the steam may enter and leave. Immediately outside these caps the rod should have transverse scratches. The length between these scratches is measured at the temperature t° of the air, and the readings of microscopes focussed upon them are noted. These microscopes being rigidly fixed, steam at 100° is blown through the tube for about ten minutes, and the microscopic readings on the scratches again noted. The difference of the original and final microscope readings is the alteration in length, and all the necessary data for calculation are thus obtained.

An almost equally satisfactory method, of greater simplicity, is to firmly clamp one end of a tube of the substance, to focus a microscope on a mark near the other end, and to pass steam through the tube. The necessary observations require no further explanation than are given above.

61.

To determine the Coefficient of Expansion of a Liquid relatively to Glass.—By this coefficient, known also as that of "apparent" expansion in glass, is understood the difference of their increase in volume per cubic centimetre at 0° for 1° rise of temperature.

A weight thermometer consists of a glass bulb drawn out into a narrow neck. The weight W_1 of this is found when empty and dry. It is then filled with the liquid by alternate heating over a flame and cooling. In the former operation extreme care is necessary to avoid fracture of the bulb. During the whole time the neck dips into the liquid. The small bubble which generally persists may be got rid of by repeatedly boiling the liquid in the bulb and cooling. The bulb, its neck still dipping into the liquid, is now cooled to the temperature of the air. After this cooling has proceeded for some time, it may be accelerated by pouring cool water over the bulb. Having arrived at the temperature required it is dried and weighed. It is very necessary not to warm the bulb by handling, and to avoid this a loop of thread may be passed over the hooked neck for the purpose of transferring it about.

Let its weight be W_2.

It is now hung over the side of a vessel filled with water, so as to be immersed with as much of the neck below the surface as possible. The water is heated to 100°, and the liquid which escapes from the neck collected in any convenient receptacle.

After remaining in the boiling water for several minutes, the bulb is withdrawn, allowed to dry, and again weighed.

Let its weight be W_3.

Now, the weight of liquid $W_3 - W_1$ has the same volume relatively to the bulb at 100° as the weight $W_2 - W_1$ has at $t°$, the temperature of the air. Therefore—

$$\frac{(W_2 - W_1) - (W_3 - W_1)}{W_3 - W_1} = \frac{W_2 - W_3}{W_3 - W_1}$$

is the proportional increase of volume between these temperatures. And—

$$\frac{W_2 - W_3}{(W_3 - W_1)(100° - t)}$$

is the apparent coefficient of expansion.

It will be observed that for strict accuracy the weight W_2 should be found when the temperature is zero.

The absolute coefficient of expansion may be obtained by adding to the above expression the coefficient of cubical expansion for glass.

62.

To determine the Coefficient of Expansion of a Liquid by a Method of Weighing.—A balance with a hole bored in its base beneath one scale-pan should be supported above the table, at such a height that a vessel of the liquid may be heated beneath the hole. A solid of known cubical coefficient of expansion, and slightly denser than the liquid, should be suspended from the pan by a thread passing through the hole. This is weighed in air, and in the liquid at the temperature of the air, after which the weighing is again performed with the liquid at some noted temperature near 100° C.

Let $t°$ be the temperature of the cool liquid.
$T°$ be the temperature of the hot liquid.
W_t be the loss of weight in the cold liquid.
W_T be the loss of weight in the hot liquid.
V be the volume of the body when at $t°$.
c be the coefficient of cubical expansion of the body.
d_t, d_T be the density of the liquid at $t°$ and $T°$.

Then—

$$W_t = Vd_t$$
$$\text{and } W_T = V\{1 + c(T - t)\}d_T$$

$$\frac{W_t - \dfrac{W_T}{\{1 + c(T - t)\}}}{\dfrac{W_T}{\{1 + c(T - t)\}}} = \frac{d_t - d_T}{d_T}$$

But the volume of a given weight of liquid is inversely as its density; hence—

$$\frac{W_t\{1 + c(T - t)\} - W_T}{W_T(T - t)} = \frac{d_t - d_T}{d_T(T - t)} = \frac{V_T - V_t}{V_t(T - t)}$$
$$= \text{coefficient of expansion}$$

63.

To determine the Coefficient of Cubical Expansion of a Solid by the Weight Thermometer.—A solid, having its weight and volume previously found, is placed in the thermometer before the neck is drawn out. The neck being drawn, the combination is weighed, and the weight of the glass found. The instrument is then filled with a liquid of which the coefficient of expansion and the density are known, and weighed at the temperature of the air. This temperature, and weight of liquid, are noted. The weights of solid and liquid, divided by their respective densities, then give their volumes at this temperature. Let the volume of liquid be V_l and that of solid be V_s; then the internal volume of the glass, $V_{ls} = V_l + V_s$. The thermometer should now be heated to 100° by being immersed in boiling water. After removal from the bath, and its exterior having become dry, it is again weighed. The difference between this and the previous weighing, divided by the density of the liquid, is the volume expelled. This equals the increase in volume of the liquid, plus the volume expelled by the expansion of the solid, minus the increase in volume of the glass. Let c_l, c_s and

c_g be the coefficients of expansion for the liquid, solid, and glass, respectively. Also let (100°—the noted temperature of the air) = $t°$. Let the volume expelled be v. Then—

$$\begin{aligned} v &= V_l(1 + c_l t) + V_s(1 + c_s t) - V_g(1 + c_g t) \\ &= V_l(1 + c_l t) + V_s(1 + c_s t) - (V_l + V_s)(1 + c_g t) \\ &= V_l(c_l - c_g)t + V_s(c_s - c_g)t \end{aligned}$$

whence—

$$c_s = \frac{v}{V_s t} - \frac{V_l}{V_s}(c_l - c_g) + c_g$$

64.

To determine the Variations in the Volume and Density of a Liquid with Change of Temperature, by Means of the Dilatometer.—The dilatometer is a narrow glass tube expanded at one end into a large bulb.

Its weight is found when empty, and again when filled with liquid to nearly the end of the tube. The position of the meniscus should then be marked or recorded when the instrument is immersed in water at 0°. To ensure the liquid being itself at 0°, it should be immersed for some time before the observation is made.

A portion of the liquid is then driven out by slightly heating the bulb, thus bringing the meniscus to within a few centimetres of the bulb when cold. Its position is again marked or recorded under the same conditions as before, and the weight again found.

In all weighings, the outside of the dilatometer should be carefully dried.

Knowing the density of the liquid at 0°, the above weighings furnish the internal volume up to the lower mark, and that of unit length of the tube.

The water around the bulb should now be slowly heated, and the height of the meniscus, relatively to the lower mark, observed by a cathetometer for every half-degree between 0°

and 6°, and for every ten degrees of temperature higher. It is well to take frequent and careful readings in the neighbourhood of 4°. The column may rise above the limits of the scale with some forms of cathetometer. When this seems about to occur, the temperature should be read, and the level of the meniscus marked on the tube. A block should then be placed under the cathetometer, and the telescope screwed down to view the mark made, after which observations proceed as before.

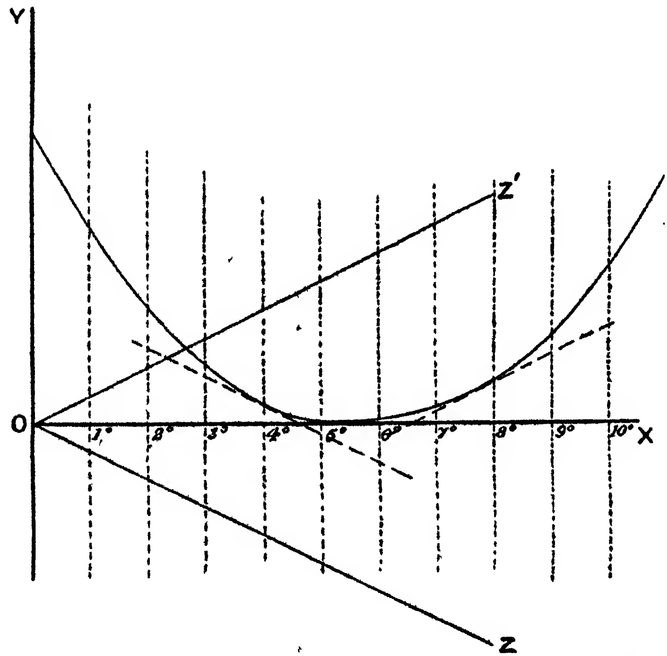
Results should be tabulated thus:

Temperature.	Height of meniscus.	Total apparent volume of liquid.

A curve should be plotted with temperatures as abscissæ and total volumes, minus the least observed volume, as ordinates to a large scale (about 1 cm. to 0·001 c.cm.). This is the curve of apparent increase of volume relatively to the glass. To obtain the absolute increase of volume, a curve is set out below OX showing the increase of volume of the bulb. If this be assumed to be uniform at all temperatures, it is a straight line, and may be set out by dropping an ordinate from any point representing a temperature T° on OX, and making it equal to V_0gT on the previous scale, V_0 being the volume of the bulb at 0°, and g being the coefficient of cubical expansion for the glass. The straight line through the O and Z, the end of the ordinate, is then the glass expansion curve. The absolute increase of volume of the liquid is the perpendicular distance between the two curves, and the temperature of minimum volume, *i.e.* of maximum density, in the case of water, is found by drawing a line parallel to OZ and tangential to the liquid curve, and noting the temperature corresponding to the point of contact. At this temperature the coefficients also are equal for both the liquid and the glass, but of opposite signs. To determine the temperature at which they are both equal and of the same sign, a line, OZ′, is drawn

above OX, making the angle XOZ′ equal to the angle XOZ, and a line drawn parallel to this and tangential to the liquid curve. This, as before, touches the latter at a point giving the required temperature.

The absolute coefficient of cubical expansion at any temperature t° is obtained by drawing a tangent to the curve there, cutting OX. If the length l cut from OX between this tangent and the ordinate of t° be expressed in degrees, and v be the volume indicated by the ordinate, V_0 the volume of the



liquid at o°, then the coefficient of cubical expansion $= \frac{v}{lV_0}$.

The difficulty in this method is the uncertainty as to the liquid in the bulb being at the same temperature as the bath. On this account the temperature of the bath should be kept at each temperature for as long a period as possible, consistent with completion of the observations in a reasonable time.

65.

To determine the Relation between the Temperature and Pressure of Air at Constant Volume, and to find the Coefficient of Increase of Pressure.—The coefficient of increase of pressure is the ratio of the increase per degree rise of temperature to the pressure at 0° C., the volume being constant. The apparatus employed is a constant-volume air-thermometer, consisting of a bulb connected to U-tubes filled with mercury, which acts as a gauge, by raising or lowering the open limb of which the mercury-level is adjusted. A vessel of water at a noted temperature should be placed around the air-bulb, and the mercury levels adjusted until the one is at a mark on the tube near the bulb, and they are level in each limb. The temperature of the water should then be raised to 40°, 60°, 80°, and 100°. At each of these temperatures the open limb should be raised until the mercury in the other is exactly at the mark, and the difference of levels in the limbs noted. In practice, it will be found better to heat the water slightly above these temperatures, and to adjust the levels as it falls to the temperature upon removal of the flame.

The total pressure on the enclosed air is in each case the atmospheric pressure, as given by the barometer, plus the difference of level in the limbs.

Results should be tabulated as follows:

Reading of closed tube.	Reading of open tube.	Difference in level.	Total pressure.	Temperature.

A curve should be plotted with absolute temperatures, *i.e.* the Centigrade temperature plus 273, as abscissæ, and total pressures as ordinates.

The coefficient of increase of pressure may be found by drawing a line parallel to the axis of ordinates through the

point corresponding to the absolute temperature of 0° C., and producing the curve to intersect this line, thus giving an ordinate for the pressure at 0° C. The mean coefficient of increase of pressure between 0° and 100° is then equal to—

$$\frac{\text{pressure at } 100^\circ - \text{pressure at } 0^\circ}{100 \times \text{pressure at } 0^\circ}$$

66.

To determine the Coefficient of Expansion of Air at Constant Pressure.—This coefficient is the ratio of the increase in volume per degree rise of temperature to the volume at 0°, the pressure being constant.

A steam heater, consisting of two coaxial cylinders, the annular space between which is closed and fitted with cocks so that steam may be passed through, is connected with a boiling-can, and the highest temperature to which it rises observed by means of a thermometer placed in the central chamber. Let this be T°.

A glass tube, somewhat longer than the heater, of a few millimetres' bore, and closed at one end, has a bead of mercury adjusted at about one-third of its length from the open end by being heated and dipped in mercury. After cooling, the tube is held vertically, open end upwards, and the length occupied by the enclosed air measured. As it is essential that at the time of measuring the air inside should be at the same temperature as that outside, the tube should be held by the top and not otherwise handled. Let l be this length and $t°$ the temperature. The tube is now placed in the heater, being held in position by passing through a cork, and adjusted until, when stationary, the bead is just seen outside the heater. The distance from its end to the top of the tube is measured, and hence L, the distance from the inner end of the bead to the closed end of the tube, calculated.

The absolute temperatures are now set out as abscissæ, the lengths as ordinates, and a straight line drawn through the two

points. The point in which this cuts an ordinate drawn at the absolute temperature 273, gives the length at 0° C. Then, if this be L_0 —

$$\left.\begin{array}{l}\text{Coefficient of ex-}\\ \text{pansion at con-}\\ \text{stant pressure}\end{array}\right\} = \frac{\text{volume at } T^\circ - \text{volume at } t^\circ}{(T^\circ - t^\circ)\text{ volume at } 0^\circ} = \frac{L - l}{(T - t)L_0}$$

67.

To determine the Water-equivalent of a Calorimeter.—The water-equivalent is that weight of water which has the same thermal capacity as the calorimeter with its stirrer and thermometer.

The calorimeter pot is removed from the outer vessel, weighed when empty and dry; then about one-third filled with cold water and again weighed. Let the weight of water contained be W_1. The pot is now replaced, and both stirrer and thermometer put in. Meanwhile water should be heated in a jacketed vessel to about 80°, its temperature being given by a thermometer inserted into the inner chamber. When this temperature is reached, that θ_1 of the cold water in the calorimeter is noted, some hot water is run out into a beaker to warm the outlet pipe, and the latter being dipped into the calorimeter, sufficient hot water is run out to nearly fill the pot. It is meanwhile necessary to note the time at which the hot water begins to flow into the calorimeter and its temperature; also the mixed hot and cool water must be well stirred.

Let θ_2 be the temperature of the hot water, and θ_3 the highest temperature recorded by the calorimeter thermometer, this last being reached T minutes after the hot water began to flow in.

The time during which the temperature falls one degree is now observed. Let t be this interval, and θ_4 the mean temperature during the time. Then at this temperature the

rate of cooling is $\frac{1}{t}$ degrees per minute. Now, the rate of cooling is proportional to the excess of the calorimeter's temperature above that of the surrounding air. Therefore, if θ_1, the temperature of the original cold water in the calorimeter, is the same as that of the air, the mean rate of cooling during the interval T was—

$$\frac{1}{t} \cdot \frac{\theta_3 - \theta_1}{2(\theta - \theta_1)} \text{ degrees per minute}$$

and consequently, to correct the observations for the loss of heat by radiation, conduction, and convection, the temperature θ_3 must be increased by the amount—

$$\frac{1}{2} \cdot \frac{T}{t} \cdot \frac{(\theta_3 - \theta_1)}{(\theta - \theta_1)}$$

Now, denoting this corrected value by θ_4, and the water equivalent of the calorimeter by WEq, the hot water has lost an amount of heat equal to—

$$W_2(\theta_2 - \theta_3)$$

where W_2 denotes the weight of it run into the calorimeter, and determined by weighing the pot a third time.

The cool water has gained an amount equal to—

$$W_1(\theta_4 - \theta_1)$$

and the calorimeter an amount equal to—

$$WEq(\theta_4 - \theta_1)$$

Equating these—

$$(WEq + W_1)(\theta_4 - \theta_1) = W_2(\theta_2 - \theta_3)$$

$$WEq = W_2\frac{\theta_2 - \theta_3}{\theta_4 - \theta_1} - W_1$$

The water may with sufficient accuracy be measured instead of weighed, weights in grams and volumes in cubic centimetres being regarded as numerically identical.

Concordant results are difficult to obtain experimentally, and it is more satisfactory for practical purposes to calculate the water equivalent as the sum of—

(*a*) The weight of the calorimeter pot and stirrer multiplied by the specific heat of their material.

(*b*) The estimated weight of mercury in the thermometer multiplied by the specific heat of mercury.

(*c*) The weight of glass thermometer immersed multiplied by the specific heat of glass.

68.

To determine the Specific Heat of a Solid Insoluble in Water.—The specific heat of a substance at any temperature is the ratio of the amount of heat required to raise unit weight of it from half a degree below to half a degree above that temperature, to the amount required to raise unit weight of water at 0° C. one degree in temperature. The specific heat of nearly all substances varies slightly with the temperature, but such variations may usually be neglected.

The thermal capacity of a body is the amount of heat required to raise its temperature by one degree, and equals its weight multiplied by its specific heat.

The principle involved in the method here explained is that a quantity of heat may be distributed in various ways, but remains unchanged in total amount unless converted into other forms of energy.

A calorimeter of known weight and water equivalent is about two-thirds filled with water at the temperature of the air and again weighed, after which it is replaced within the outer pot.

The solid, which should be of such shape, where possible, as to possess a large surface relatively to its volume, is weighed, and suspended inside a steam heater by a thread. This thread

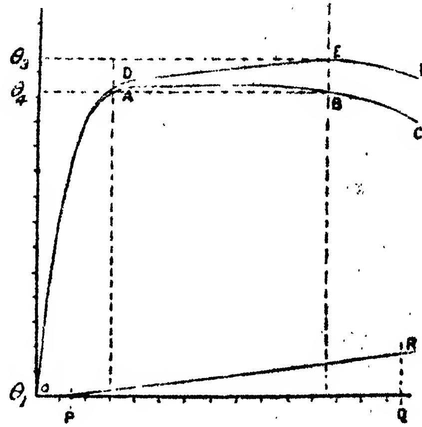
should be sufficiently long to allow of the solid being lowered through the heater into the calorimeter, and while the heating proceeds, it is, together with a thermometer, grasped by a cork, which closes the top of the heating chamber. The bulb of the thermometer should be near to the solid.

When the latter has been heated for ten minutes or longer, and has acquired the temperature of the heater, the thermometers in the calorimeter and heater are read. The calorimeter is now placed momentarily under the heater, and the solid is gently, but quickly, lowered into it. The water is well stirred, and the temperature at the end of the first and each succeeding half-minute noted. At first the temperature rises rapidly, but later remains stationary, the heat given out by the solid during this period being equal to that lost by radiation, etc. Observations should continue beyond this stage until the temperature has fallen one or two degrees. The curve OABC should now be set out with times as abscissæ and the rise of temperature as ordinates. The calorimeter should now be emptied, filled to the previous level with hot water, and its rate of cooling per half-minute at some high temperature noted by a sensitive thermometer. This may be, of course, more accurately determined by taking one-fourth of the temperature drop in an interval of two minutes. Consider the rate so found to be t° per half-minute when the water is T° warmer than the surrounding air. Then, if the water were θ° warmer than the air, the rate would be $\frac{\theta}{T}t$, since the rate is proportional to excess of temperature.

Let now P be taken midway between O and the foot of that ordinate at which the curve bends over, and let the line PR be drawn with PQ equal to any number n of half-minutes, QR equal to $\frac{n\theta t}{T}$ degrees, where θ is the temperature rise corresponding to the horizontal portion of the curve.

Then, with sufficient accuracy, the highest temperature rise which would exist if there had been no loss of heat by radiation, convection, etc., is given by the greatest ordinate of the curve ODEF, obtained by adding the ordinates of OABC and PR.

Let this be $(\theta_3 - \theta_1)$, where θ_3, θ_1 are the theoretically final and the original temperatures of the water. Let also θ_2 be the temperature of the heater, W the weight of water, w the weight



of the solid, WEq the water equivalent of the calorimeter, and σ the specific heat of the solid. Then, considering the specific heat of water as unity at all temperatures—

$$\text{Loss of heat from solid} = w\sigma(\theta_2 - \theta_4)$$

where θ_4 is the ordinate of the first curve corresponding to θ_1 on the last.

$$\text{Gain of heat by calorimeter and water} = (W + WEq)(\theta_3 - \theta_1)$$

These are equal; hence—

$$w\sigma(\theta_2 - \theta_4) = (W + WEq)(\theta_3 - \theta_1)$$

$$\sigma = \frac{(W + WEq)(\theta_3 - \theta_1)}{w(\theta_2 - \theta_4)}$$

69.

To determine the Specific Heat of a Liquid.—A calorimeter of known water-equivalent, WEq, is weighed when

empty and dry; subsequently when about two-thirds filled with the liquid. A solid of noted weight and known specific heat is raised to a high temperature in a steam heater; this temperature must, however, be below the boiling-point of the liquid. The subsequent operations are precisely as described in Art. 68.

The loss of heat from the solid $= w\sigma(\theta_2 - \theta_4)$

The gain of heat by the calorimeter and liquid $\Big\} = (W\sigma' + WEq)(\theta_3 - \theta_1)$

where W is the weight of the liquid, σ' its specific heat. These are equal; hence—

$$w\sigma(\theta_2 - \theta_4) = (W\sigma' + WEq)(\theta_3 - \theta_1)$$

$$\sigma' = \frac{w\sigma(\theta_2 - \theta_4)}{W(\theta_3 - \theta_1)} - \frac{WEq}{W}$$

70.

To determine the Specific Heat of Ice.—The inner pot of a calorimeter is weighed when empty, and again when about half filled with some oil for which the specific heat is known. It is then surrounded with a freezing mixture until its temperature has fallen as far below zero as possible, after which it is removed, wiped dry externally, and placed within its outer pot. Its temperature is now noted at intervals until it has risen by one or two degrees, both times and temperatures being recorded. Small pieces of ice at $0°$ are wiped dry with flannel and dropped into the well-stirred oil, piece by piece, until its temperature is about a degree below zero. During this operation, the observations of time and temperature have continued, and are maintained until the mixed oil and ice are at $0°$.

The weight of ice added is found by again weighing the inner pot and its contents.

The rise of temperature produced in the oil by the ice is found by setting out a curve with times as abscissæ and temperatures as ordinates. An abrupt change in the curve occurs

when the ice is added, but after all the ice has cooled to the temperature of the oil around it, the curve resumes very nearly its original direction. If the earlier part of the curve be produced, the maximum vertical distance between it and the later portion is the rise of temperature produced in the oil, and the point on the curve where this distance exists, gives by its ordinate the common temperature of oil and ice after the latter's cooling.

Let the weight of oil be W, its specific heat be σ', and its rise of temperature be T'. Let WEq denote the water equivalent of the calorimeter, w the weight of ice added, T its fall of temperature, and σ its specific heat. Then the ice has lost an amount of heat equal to $w\sigma T$. The oil and calorimeter have gained an amount—

$$(W\sigma' + WEq)T'$$

But these quantities are equal; therefore, equating them together—

$$w\sigma T = (W\sigma' + WEq)T'$$

$$\text{or } \sigma = \frac{T'}{T} \cdot \frac{(W\sigma' + WEq)}{w}$$

71.

To determine the Specific Heat of a Liquid by the Method of Cooling.—The rate at which heat is radiated from a body depends only on the nature and area of its surface and the difference of temperature between itself and surrounding objects, being proportional to this difference if it be small.

Consequently, if there be two heated bodies cooling under the same conditions of surface and temperature, since each loses heat at the same rate, the temperature of that having the smaller thermal capacity falls the more rapidly, and the time taken by each to fall through the same range of temperature is proportional to its thermal capacity. Hence—

$$\frac{T_1}{T_2} = \frac{\text{thermal capacity}_1}{\text{thermal capacity}_2} = \frac{\text{weight}_1 \times \text{sp. heat}_1}{\text{weight}_2 \times \text{sp. heat}_2}$$

Therefore—

$$\frac{\text{sp. heat}_1}{\text{sp. heat}_2} = \frac{T_1 \times \text{weight}_2}{T_2 \times \text{weight}_1}$$

A convenient form of apparatus consists of a jar in which is a large boiling tube which may be supported by the cork. This tube is closed by a cork through which passes a smaller tube containing the liquid under investigation.

The outer space between the jar and tube is filled with cold water, and the innermost tube, half filled with the liquid, is heated to about 90°. The volume of liquid contained is noted, and the tube is then inserted as centrally as possible in the larger one. A loosely fitting cork through which a thermometer passes should be inserted in the tube immediately after heating, and by means of the latter the exact time of cooling from 80° to 60° should be noted. Thus T_1 is found.

The weight of the tube and liquid is now determined, after which the latter is poured out, an equal volume of water substituted, and all operations repeated. Finally, the empty tube is dried and weighed. To preserve equality of conditions the water in the outer jar should be renewed with fresh, at the same temperature as the first, before the latter half of the determination.

Since in this case water, for which the specific heat is unity, is employed, that of the liquid may be found. For greater accuracy the thermal capacity of the tube should be allowed for. This may be found by multiplying the weight of glass tube below the level of the liquid's surface by its specific heat as found from tables. The corrected expression then becomes—

$$\text{Specific heat of liquid} = \frac{T_1 \text{ weight}_2 + (T_1 - T_2)c}{T_2 \text{ weight}_1}$$

where c denotes the thermal capacity of the tube and the immersed portion of the thermometer, T_2 the time of the cooling with water.

72.

To determine the Specific Heat of a Substance by Means of the Steam Calorimeter.—The steam calorimeter consists of a balance supported above a chamber into which wires pass to support a second pair of pans inside it. This chamber may, when desired, be filled with steam at 100°.

The balance-pans being perfectly dry, the body of which the specific heat is required is placed in one pan, counterbalanced, and its weight noted. Steam, having been meanwhile generated, is now allowed to rush into the chamber. A sufficient weight of steam condenses on each pan to raise the temperature to 100°. In addition, a further amount condenses in the one pan raising the temperature of the body to 100°, and also disturbing the equilibrium of balance. Let this be restored after a few minutes by adding noted weights to the external pan. If the balance so obtained is permanent, the body has acquired a temperature of 100°, and the steam is turned off.

Let t° = the original temperature of the body.
W = the weight of the body.
σ = the specific heat of the body.
L = the latent heat of steam.
w = the weights added to restore balance.

The heat absorbed by the body is equal to $W\sigma(100 - t)$.

The heat given out by the water condensed on the body is equal to wL. Hence—

$$\sigma = \frac{wL}{W(100 - t)}$$

In practice, to prevent the accumulation of water at the holes where the wires enter, by reason of the steam condensing, the suspension wires are there encircled by small coils of high resistance wire, which may be heated to redness by an electric current. This should be switched on just previously to admitting steam.

73.

To determine the Specific Heat of Air at Constant Pressure.—An accurate determination of this quantity is, owing to the low density of gases, impossible in the absence of elaborate apparatus. An approximate value may, however, be obtained by passing air from a pump through a coiled tube immersed in a hot bath, and afterwards through another surrounded by cold liquid in a calorimeter.

The volume of air discharged by a condensing syringe or a bicycle pump is approximately constant for a full stroke, and may be determined by leading the discharge tube under a graduated vessel arranged for the collection of gases by upward displacement of water. Let the volume so collected be observed, and corrected to the pressure of the atmosphere at the time.

This is the volume driven out per stroke of the pump, and from it the weight of air per stroke may be calculated.

The heating bath may be filled with either water or oil, the latter permitting the temperature to be above 100°. The calorimeter may advantageously be filled with oil of a known specific heat—so giving a greater rise of temperature than water for the same amount of heat absorbed. The temperature of the heater and calorimeter being noted, air is driven slowly through by a considerable known number of pump-strokes, and the rise of temperature in the calorimeter observed.

Let WEq = the water equivalent of the calorimeter.
W = the weight of liquid in the calorimeter.
σ' = the specific heat of the liquid.
T = the constant temperature of the hot bath.
t_1 = the original temperature of the calorimeter.
t_2 = the final temperature of the calorimeter.
w = the weight of air discharged by one pump stroke.
N = the number of pump-strokes made.

Then the average final temperature of the air is $\frac{t_1 + t_2}{2}$, and the specific heat of the air is equal to—

$$\frac{(W\sigma' + WEq)(t_2 - t_1)}{Nw\left(T - \frac{t_1 + t_2}{2}\right)}$$

74.

To determine the Ratio between the Specific Heats of Air at Constant Pressure and at Constant Volume by the Method of Clement and Desormes.—The ratio $\frac{\text{sp. heat at constant pressure}}{\text{sp. heat at constant volume}}$, commonly denoted by γ, may be ascertained by means of a large vessel, into which air may be compressed by a pump. The addition of a pressure gauge and a tap or valve of large aperture, by which the air may be released, is also necessary. The gauge is usually an open U-tube, containing sulphuric acid or oil.

Air is firstly compressed into the vessel by the pump, passing through drying tubes before entering, until a pressure of 20 or 30 cms. of acid is shown. After some five or ten minutes, during which the air arrives at the temperature of the external atmosphere, the gauge is read. Let the volume and pressure of the enclosed air be denoted by p_1, v_1. The valve is now opened for about a second and the gauge reading noted. The pressure and volume have become p_2 and v_2, connected with the previous values by the adiabatic relation—

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the air has become cooled.

The gauge is carefully watched, and the greatest pressure which it creeps up to as the enclosed air regains its original temperature by conduction noted. Let this be p_3. If the expansion had occurred by the vessel expanding to v_2, it is clear that the volume and pressure would be v_2, p_3. Without

affecting the conditions, this may be supposed to have been the case so far as the air in the vessel is concerned. This is again at its original temperature; hence, by Boyle's law—

$$\therefore p_1v_1 = p_3v_2$$

$$\log v_2 - \log v_1 = \log p_1 - \log p_3 \quad . \quad . \quad (2)$$

and from (1)—

$$\frac{\log p_1 - \log p_2}{\log v_2 - \log v_1} = \gamma$$

Thus from (2)—

$$\frac{\log p_1 - \log p_2}{\log p_1 - \log p_3} = \gamma$$

It is to be noted that p_1, p_2, p_3, indicate the total pressure, *i.e.* (gauge reading) $\times \dfrac{\text{density of acid or oil}}{\text{density of mercury}} +$ height of barometer. If the pressure, p_1, is only a little greater than that of the atmosphere, the numbers representing p_1, p_2, p_3, may be used instead of their logarithms, and in this case the barometer reading may be omitted, the difference of gauge levels being taken in the equation.

If when the valve is opened the pressure of the air falls to that of the atmosphere, p_2 is zero by the gauge, and its absolute value may be noted from the barometer. The escape of air, however, is not "dead-beat," but oscillatory; it is, therefore, necessary to ascertain by means of smoke produced near the valve, or other methods, the period of oscillation. The valve may then be opened for just the half-period. It is preferable to open it for somewhat less, so letting p_2 have some small gauge reading.

In general, the fluid in the gauge will either lag or oscillate by reason of inertia; during the period of its coming to rest, the air within the vessel has been abstracting heat from its walls, as a result of which the pressure then recorded is somewhat greater than the adiabatic p_2.

To correct for this, the time of opening the valve, the gauge reading when first steady, and at every subsequent half-minute,

should be observed. A curve of times, and pressures—the former as abscissæ, the latter as ordinates—may be set out, and the value p_2, when the valve was opened, found by extrapolation.

75.

To determine the Latent Heat of Water.—The latent heat of water is the quantity of heat required to convert one gram of ice at 0° to water at the same temperature.

A calorimeter of known water-equivalent, WEq, is weighed empty and dry, also when about two-thirds filled with water at about 80°. After being replaced in its outer pot, the temperature is noted and the time taken. Immediately upon doing this, small pieces of ice, previously broken, are wiped dry and gently dropped into the water. This is continued, with constant stirring, until the temperature has fallen to about 10°. All the ice being fused, the temperature and time are again noted. The calorimeter is then weighed, emptied, and refilled with hot water, that a determination of the rate of cooling may be made as in Art. 68. Let this be $t°$ per minute, with an excess of temperature T.

Let a weight W_1 grams of water at θ_1 be in the calorimeter before adding ice, W_2 be the weight of ice added, and θ_2 be the final temperature.

If θ be the temperature of the air, the loss of heat by radiation and convection has been approximately—

$$(W_1 + WEq)\left(\frac{\frac{\theta_1 + \theta_2}{2} - \theta}{T} tN\right)$$

where N is the difference of the observed times in minutes.

$$\left.\begin{array}{l}\text{The heat lost altogether by the calori-}\\ \quad\text{meter and hot water}\end{array}\right\} = (W_1 + WEq)(\theta_1 - \theta_2)$$

$$\text{The heat absorbed by the ice in fusing} = W_2L$$

$$\left.\begin{array}{l}\text{The heat absorbed by the cold water}\\ \quad\text{from the ice}\end{array}\right\} = W_2(\theta_2)$$

Hence—

$$(W_1+WEq)(\theta_1-\theta_2)=W_2(L+\theta_2)+(W+{}_1WEq)\left(\frac{\theta_1+\theta_2-2\theta}{2T}tN\right)$$

$$(W_1+WEq)\left\{\theta_1\left(1-\frac{tN}{2T}\right)-\theta_2\left(1+\frac{tN}{2T}\right)+\frac{tN\theta}{T}\right\}=W_2(L+\theta_2)$$

$$\left.\begin{matrix}\text{Latent}\\\text{heat}\end{matrix}\right\}=L=\frac{W_1+WEq}{W_2}\left\{\theta_1\left(1-\frac{tN}{2T}\right)-\theta_2\left(1+\frac{tN}{2T}\right)+\frac{tN\theta}{T}\right\}-\theta_2$$

76.

To determine the Latent Heat of Steam.—The latent heat of steam is the amount of heat required to convert one gram of water at 100° into steam at the same temperature. This varies with the temperature at which boiling occurs, but, unless specially stated, refers to steam produced at 100°.

A calorimeter of known water-equivalent, WEq, is weighed when empty and dry, again when nearly filled with cold water. Steam is then passed through a trap into the water, the temperature being noted originally and at every subsequent half-minute till it reaches about 80°. The curve of times and temperatures is then set out as in Art. 68.

The water being at about 80°, the rate of cooling per half-minute is noted, and a right-angled triangle set out with the base equal to the difference of temperature between the water and surrounding air, the perpendicular equal to the rate of cooling per half-minute. From the curve the mean temperature excess for each half-minute is observed. Let these be ϕ_1, ϕ_2, ϕ_3, etc., for the first, second, third, etc., half-minutes. From the acute angle, let ϕ_1, ϕ_2, etc., be measured horizontally along the base of the triangle, and verticals be raised at each to cut the hypothenuse. The lengths of these represent the cooling per half-minute for their respective temperature excesses, to the scale of the vertical side. Let these be denoted by t_1, t_2, t_3, etc. Then the curve of temperatures existing, if no cooling by radiation had occurred, is obtained by adding to the first ordinate of the original curve a length t_1; to the second a length t_1+t_2; to the third, $t_1+t_2+t_3$, etc.

If now θ_3 is the temperature corresponding to the maximum ordinate of this curve, θ_1 the original temperature of the cold water, and θ_4 the temperature on the first curve corresponding to θ_3 on the last, W the weight of cold water, w that of steam—

The loss of latent heat from steam $= wL$
The loss of heat from condensed water $\} = w(100 - \theta_4)$
Gain of heat by water plus loss by radiation and conduction $\} = (W + WEq)(\theta_3 - \theta_1)$

Hence—

$$w\{L + (100 - \theta_4)\} = (W + WEq)(\theta_3 - \theta_1)$$

$$\text{Latent heat} = L = \frac{(W + WEq)(\theta_3 - \theta_1)}{w} - (100 - \theta_4)$$

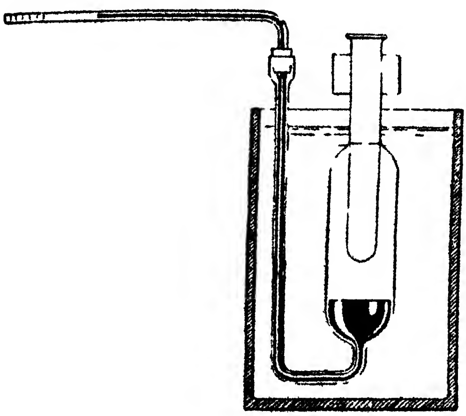
77.

To determine Specific Heats by Means of Bunsen's Ice Calorimeter.—This calorimeter consists of a bulb connected below to a vertical tube, and having a tube projecting into its interior from above. The tube with which the bulb communicates should be expanded at the top so that a cork may be inserted. A separate tube is passed through the cork, and having its upper part bent at right angles, acts as a gauge. This upper horizontal portion should be of a small, but uniform, bore, and may advantageously be calibrated by the method of Art. **10**, and graduated in millimetres. In any case, its average internal volume per unit length must be determined by filling with mercury as there described.

The bulb and adjoining tube are completely filled with recently boiled water by alternate heating and cooling, after which mercury is poured in, displacing part of the water and filling the tube. The gauge-tube is also completely filled with mercury up to its bend, and fixed by means of the cork in connection with the other part.

This being done, and the temperature of the water having

fallen nearly to that of the external air, the bulb is surrounded by a freezing mixture. A small quantity of ether is poured into the tube projecting into the bulb, which may be termed the test-tube, and evaporated rapidly by having air bubbled through it. This causes part of the water to freeze, thereby expanding and driving some mercury into the gauge-tube. An appreciable quantity of ice having formed, the ether is drawn, or sucked by a roll of blotting-paper, from the test-tube, and more air blown in to dry it. This should be done



as soon as the mercury in the gauge comes within a centimetre or two of its open end.

Readings are now taken at brief intervals for some ten minutes of the mercury's position in the gauge, the positions and times being recorded.

The maximum temperature reached by the interior of a steam heater is noted, and the body, of which the specific heat is required, heated in it to that temperature. This body must be of such size and shape that it may be contained in that portion of the test-tube surrounded by the bulb. Sufficient water at 0° to immerse the body is then poured into the test-tube, and the heated body lowered into it, the open end of the tube being immediately closed by a plug of cotton-wool. The

gauge readings, which have been continuously recorded, with their times, until the dropping in of the body, now rapidly diminish owing to the fusion of some of the ice, and observations continue as previously, until the steady increase again occurs at about its original rate.

During the cooling of the body a differential process has gone on, the internal body giving, and the external freezing mixture abstracting, heat. To determine the amount of ice liquefied by the heat of the body, a curve is set out having times as abscissæ and gauge readings as ordinates. Until the time of the body being dropped in this is a straight line, but it then falls, ultimately rising and proceeding approximately parallel to its original direction. If the original part be produced, the maximum distance vertically between it and the actual curve represents the alteration in the gauge reading produced by the heated body. The volume per unit length of the gauge being known, the volume change produced in the ice and water is found. Now, 1 gram of ice in liquefying is reduced 0.0907 c.cm. in volume, so that 1 calorie, fusing $\frac{1}{80}$ gram of ice, produces a volume change of 0·00113 c.cm. Hence, if the volume change, as found above, be V, the temperature of the heater be T, the weight of the body be W, and its specific heat be σ—

$$WT\sigma = \frac{V}{0{\cdot}00113}$$

$$\sigma = \frac{V}{WT \times 0{\cdot}00113} = 885\left(\frac{V}{WT}\right)$$

78.

To determine the Maximum Vapour-pressures for Water at Temperatures between 80° and 100°.—Liquids boil when their temperature is such that the maximum vapour-pressure for that temperature is equal to the atmospheric or other pressure on the surface.

Hence the relation between maximum vapour-pressure and

temperature may be found by varying the pressure of the air over a liquid and observing the corresponding boiling-point.

A boiler fitted with thermometers is employed. This should be connected, through a condenser and air-vessel, to an air-pump and a gauge. The condenser is necessary, as without it the pressure would increase and boiling cease. The object of the air-vessel is to minimize perturbations of pressure.

Air should be exhausted by the pump until a difference of about 35 cms. exists in the levels of a mercury gauge. The pump-tube should then be tightly clipped. Cold water is now sent through the condenser, and the boiler heated. The highest temperature reached by the boiler's thermometers is the boiling-point.

This and the gauge-levels should be observed together.

A little air is then allowed to enter by slightly loosening the pump-tube clip, after which the new temperature and boiling-points are again noted. Observations should be made for six or more pressures, and the results tabulated thus:

Temperature.	Difference of gauge-levels.	Vapour-pressure.

The total pressure is the barometric height minus the difference of gauge-levels, assuming an open U-tube gauge.

A curve should be plotted with temperatures as abscissæ and pressures as ordinates.

By compressing air into the boiler, values of the maximum vapour-pressure may be found for temperatures above 100° in a similar manner.

79.

To determine the Maximum Vapour-pressure of Water at Temperatures between 0° and 80°.—The apparatus employed usually consists of a barometer tube surrounded at the

upper part with a large water-jacket, having a window through which the level of the mercury in the tube may be observed.

The tube is firstly filled with mercury and inverted in a small trough of the same.

By means of a bent tube with an indiarubber cap, small quantities of water are then allowed to rise up the tube and vaporize in the space above the mercury.

As soon as the vapour exerts its maximum pressure a permanent layer of water covers the mercury.

In commencing, the water in the jacket should be brought to zero by the addition of ice, cathetometers focussed on the internal and external mercury levels, and the positions noted.

Sufficient water to saturate the space should then be introduced by very small amounts at a time. When saturated, the cathetometers are again read.

The differences between the final and first readings are the changes of level, and the change of level inside minus that outside the tube is the vapour-pressure.

The water in the jacket now has its temperature raised to 10°, 15°, 20°, etc., as indicated by a thermometer immersed in it, and the maximum pressure for each temperature is found by passing in water and reading the cathetometers as previously.

Results should be tabulated thus:

Temperature.	Mercury-level inside.	Mercury-level outside.	Total pressure

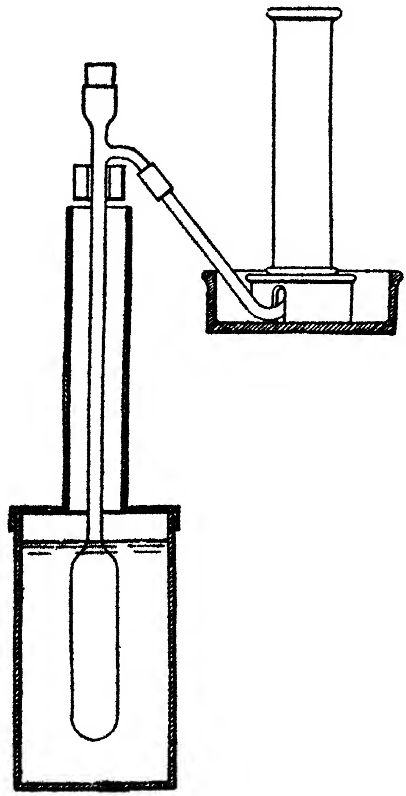
A curve should be plotted with temperatures as abscissæ and maximum vapour-pressures as ordinates.

80.

To determine the Density of a Vapour by Victor Meyer's Method.—In this method a quantity of air, equal in volume and temperature to the vapour produced from a known weight of the liquid, is removed and measured.

A small stoppered tube is weighed when empty, and again when filled with the liquid for the vapour of which the density is to be found. Thus the weight of vapour which will be produced is found.

A tube expanding into a large bulb below, and furnished



with a side branch near the top, has some asbestos-wool inserted into the bottom of the bulb, and is closed by a cork at the top.

It is then surrounded by a hot-water or steam jacket, and heated to 108°.

After remaining some time in the jacket to ensure it being at this temperature, a vessel of water, a beehive trough, and a graduated tube filled with water and inverted, are adjusted to the side branch.

The cork is then withdrawn, the small tube dropped in, and the cork quickly replaced.

The liquid now volatilizes, driving out the stopper from the small tube, and displacing its own volume of air, which passes over into the graduated tube.

This volume of air is at the atmospheric temperature and under a pressure less than that indicated by the barometer, by an amount equal to the difference of water-level inside and outside the tube, divided by the density of mercury.

Assuming the vapour to follow the laws of a perfect gas, the known weight of it would have the same volume as the air displaced under the same conditions. Hence its density, *i.e.* the weight of 1 c.cm. at 0° and 760 mm. may be found.

For greater accuracy the pressure of the air in the tube, as calculated above, should be diminished by the maximum vapour-pressure of water at the atmospheric temperature.

81.

To determine the Dew-point by Regnault's Hygrometer. —The aqueous vapour in the atmosphere is usually insufficient to saturate it at its actual temperature; the temperature at which the quantity present would just saturate it is called the "dew-point." If an object at, or below, this temperature be placed in the air, moisture will be deposited upon it.

Regnault's hygrometer consists of a glass tube, the lower part of which is furnished with a polished silver cup. This tube is closed by a cork, through which pass a thermometer and two glass tubes, one reaching nearly to the bottom, the other terminating near the top. Ether being placed in the silver cup, the shorter tube is connected to a filter-pump or aspirator. By this means a rapid stream of air-bubbles is drawn through the ether, producing a considerable reduction

of temperature. When a film of moisture appears upon the silver, the temperature is noted from the thermometer, which has its bulb in the ether. Bubbling is then stopped, and the temperature at which the film vanishes also noted. This operation is repeated several times, and the mean temperature is taken as the dew-point. The observer should not approach the instrument too closely in making observations, as respiration affects the local humidity.

Now, a saturated vapour exerts its maximum vapour-pressure; therefore the pressure of the vapour in the air is the maximum vapour-pressure for the dew-point temperature. The "humidity" of the atmosphere is the ratio of the vapour actually present, to that required to saturate it, or—

$$\text{Humidity} = \frac{\text{quantity saturating air at dew-point}}{\text{quantity saturating air at actual temperature}}$$

$$= \frac{\text{maximum vapour-pressure at dew-point}}{\text{maximum vapour-pressure at actual temperature}}$$

These temperatures being noted, the corresponding maximum vapour-pressure may be found from tables.

82.

To determine the Law of Cooling for Temperatures between 20° and 100°.—If one face of a thermopile be maintained at a constant temperature, conveniently that of the air, and the other be directed toward a heated object, the current through a galvanometer connected with it is proportional to the rate at which the face receives radiant heat. The galvanometer deflections, therefore, if small, are proportional to the rates of radiation, *i.e.* of cooling, in the heated object.

Water is heated in a Leslie's cube to 100° C., and the thermopile placed about six inches away, facing one side squarely, but carefully protected from direct radiation from the flame. Upon the water reaching 100°, the flame is extinguished, and the deflection noted.

Observations are now made every minute of the deflection, and of the temperature as given by a thermometer immersed in the cube.

After a time, when the rate of cooling has become slower, longer intervals than a minute may be taken.

Results should be tabulated thus:

Time.	Temperature.	Deflection.	$\frac{\text{Deflection}}{\text{Temp.} - \theta}$

In the last column θ denotes the temperature of the atmosphere.

Two curves, one (A) having time as abscissæ and (temperature $-\theta$) as ordinates, the other (B) having time as abscissæ and deflections as ordinates, should be set out, and their similarity observed. From the values in the last column of the table, the law should be deduced.

83.

To compare the Heat Emissivities of Various Surfaces.—The emissivity of a surface is the amount of heat radiated from unit area per second when its temperature is one degree above that of surrounding objects. It varies with the temperature, but may be considered as constant for a range of 0° to 100°.

The surfaces to be compared usually form the four vertical sides of a metal pot, termed a "Leslie's cube," in which water may be boiled, thus bringing the whole to a common temperature.

The current through a galvanometer connected to a thermopile is proportional to the difference of temperature between the faces of the latter. Hence, if one face be kept at the temperature of the air, and the other be placed about

6 inches from the cube, squarely facing one side, the galvanometer deflections, if small, are proportional to the rate at which heat is radiated from the cube.

Thus, if the cube be maintained at 100°, the emissivities of the four sides may be compared by turning it so that each side is successively at the same distance from, and squarely opposite to, the thermopile, and noting the deflection in each case.

The thermopile should be carefully screened from all direct radiation from the heating flame.

One face of the cube should be lamp-blacked, and the ratios of the radiation from the other sides to that from this side found.

84.

To compare the Intensities of Heat Radiation from Different Sources.—The radiation from the heated body or flame is allowed, in part, to pass through holes in two screens, and subsequently fall upon one face of a thermopile.

The screen next to the source of heat should be of bright metal, and as near to the source as is possible without its becoming excessively heated; the other should be some few inches distant and of thick wood. Behind the latter is the thermopile, which may advantageously be fitted with a tube of paper blackened inside. The source should be of sufficient size to rather more than cover the holes; and the face of the thermopile, the screens, with, as far as possible, the radiating surface, should lie in parallel planes. The various sources should be successively placed at the same distance from the screen, and the deflections given by the galvanometer connected with the thermopile in each case noted. The intensities of radiation are proportional to the deflections. In this way those of a flat coal-gas flame, a lamp-blacked surface at 100°, strips of metal heated by a flame behind them, etc., should be compared.

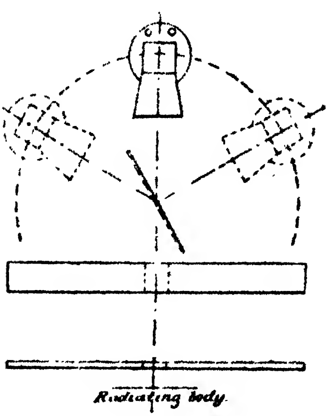
In comparing sources of widely different intensities, it may be necessary to use a smaller hole for that having the greater intensity. When this is done, the deflection must be multiplied

by the ratio of the area of the larger to that of the smaller hole to obtain its value relatively to that deflection given by the weaker source with the larger hole.

85.

To determine the Coefficient of Reflection of Various Surfaces for Heat from Various Sources.—The coefficient of reflection, or reflecting power, of a surface for heat, is the ratio of the reflected to the incident intensity. This varies according to the temperature of the source and the angle of incidence.

The general arrangement of the apparatus employed is as in the last article. The distance, however, between the thermopile and wooden screen should be somewhat greater. The direct radiation is firstly allowed to fall upon the thermopile, and the deflection noted. This being done, the reflecting surface is placed between the thermopile and screen, and adjusted to an angle of $90^\circ - \theta^\circ$ with the line passing through the holes, θ° being the angle at which the reflecting power is required. During this adjustment, radiation should be prevented from falling on the screen, either by extinguishing the source of heat, or putting a shutter between the holes.



The thermopile is now turned so as to receive the radiation from the reflecting surface, its distance from the surface being unaltered. The radiation is allowed to fall upon, and be reflected from, the surface, and the thermopile indication is again noted. This deflection is due to reflection and radiation from the surface, which itself becomes warmed. The latter effect can be corrected for by turning the thermopile through 180°, and reading

the deflection given by radiation from the back of the surface. Let the direct deflection be θ_1, that due to reflection and radiation be θ_2, and that due to the last alone be θ_3. Then—

$$\text{Reflecting power} = \frac{\theta_2 - \theta_3}{\theta_1}$$

86.

To determine the Diathermancy of Various Substances. —The diathermancy of a substance is the ratio of the heat transmitted through unit thickness to the heat entering the front face. The latter quantity is equal to the total incident heat minus the amount reflected.

The source of heat is placed behind a screen in which is a hole through which radiation passes. This radiant heat is allowed to fall upon a thermopile or radio-micrometer, and the resulting galvanometer deflection θ_1 is noted. A plate of the substance of which the diathermancy is required is then placed between the hole and thermopile, and the deflection again observed. Let this be θ_2.

These observations should be made employing various sources of radiation, and the thickness d of the plate is finally measured.

If now the coefficient of reflection, as determined for a small angle of incidence by the method of the previous article, be denoted by c, the ratio of the emergent to the entrant heat is—

$$\frac{\theta_2}{\theta_1(1 - c)}$$

and, making the usual assumption that the diathermancy is independent of the intensity of radiation—

$$I_2 = I_1 k^d$$

$$\text{or } \log_e k = \frac{\log_e I_2 - \log_e I_1}{d} = \frac{\log_e \theta_2 - \log_e \{\theta_1(1 - c)\}}{d}$$

where I_1, I_2 denote the intensities of the entrant and emergent heat, and k indicates the diathermancy.

In the case of a liquid a containing cell with plane-glass ends must be employed, θ_1 being taken when the heat passes through the empty cell, and θ_2 after filling.

Readings of the deflection should be taken as soon as the spot comes to rest, for a subsequent creeping up occurs, owing to the transmitting substance becoming warm and itself directly radiating.

87.

To compare the Conductivities for Heat of Different Substances by observing the Temperature Gradient along Bars. — The conductivity of a substance for heat at any particular temperature is defined as the number of heat units that pass per second through unit area of a plate of unit thickness, having its sides respectively maintained half a degree above, and half a degree below, that temperature.

To make determinations directly on plates is, however, difficult, and productive of inaccurate results.

Comparisons may be made with moderate accuracy between the conductivities of substances by means of bars of each similar in all other respects.

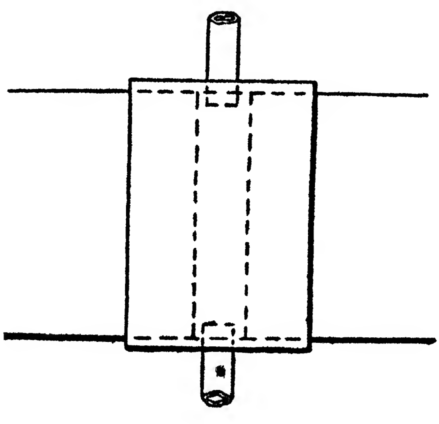
To obtain similarity of surface, the bars should be covered with paper, blacklead, etc., or electro-plated.

One end of each is heated, and when they have attained a thermally stationary state, the temperatures at three corresponding points along each bar are noted.

For the purpose of indicating temperature, small holes are sometimes drilled, into which the bulbs of small thermometers are inserted, the space between the bulb and the sides of the hole being filled with mercury. It is, however, preferable to employ a thermo-electric couple for the indication of temperature. One of platinum and platinum-rhodium is most satisfactory. This, after being standardized, may be inserted in holes 1 or 2 mms. in diameter drilled into the bars. To

standardize the couple, the wires should be connected to a sensitive low-resistance galvanometer. The deflections should then be noted, when the junction is dipped into water at different temperatures indicated by a mercury thermometer.

The temperature of the air, assumed to be the same as that of the cool ends of the couple, should also be noted, and a curve plotted with the number of degrees the water is warmer or cooler than the air as ordinates, and the galvanometer deflections as abscissæ. In order that the temperature of the cool ends may not rise, the couple should not be heated continuously, but only for brief periods. The temperature of any hole in the bar measured from that of the air may now be



learned by noticing the deflection when the couple is inserted, and referring to the curve.

The heating of the ends of the bars may be accomplished by slipping a piece of rubber tube over them, fitting two pieces of glass tubing into the rubber, and blowing steam through the narrow chamber between the ends of the bars.

The steam should pass in at the top so as to sweep out any water which condenses.

The apparatus should be set up in a spot free from draughts.

When deflections given by a thermo-couple placed near

the distant end are the same at intervals of some minutes, the condition of the bar may be taken as stationary.

Two positions on one bar at which the temperatures differ considerably are then selected, and their temperatures noted.

Let the greater of these be T_1 and the less T_3, and that of the point midway between those selected be T_2.

The temperatures of corresponding points on the other bar are now observed. Let these be T_1', T_2', T_3'.

Then, if K and K′ denote the conductivities, it may be proved that—

$$\frac{K'}{K} = \frac{\log_e\left\{\dfrac{T_1+T_3}{2T_2}+\sqrt{\left(\dfrac{T_1+T_3}{2T_2}\right)^2-1}\right\}}{\log_e\left\{\dfrac{T_1'+T_3'}{2T_2'}+\sqrt{\left(\dfrac{T_1'+T_3'}{2T_2'}\right)^2-1}\right\}}$$

It is to be noted that T_1, T_1', etc., denote the excess of the bar's temperature above that of the surrounding air.

A curve should also be drawn to represent the fall of temperature along the bar, having distances from the heated end as abscissæ, and excess of temperature as ordinates. From the point where this cuts the axis of ordinates a logarithmic curve should also be drawn for comparison.

The tangent drawn at any point on the first curve is the temperature gradient at the corresponding point on the bar to which it refers.

If, in making the preceding comparison of conductivities, the temperature falls only slightly from the heated end to the other, the outer ends of both bars should be cooled by a stream of cold water.

88.

To determine the Conductivity of a Substance by the Method of Periodic Flow.—In this method, originally employed by Angström, a long bar of the substance is surrounded at its centre by a chamber through which steam and cold water are alternately passed for equal periods. These may

conveniently be of about ten minutes' duration, the period of a complete thermal cycle then being twenty minutes. After several such alternations, if the bar be protected from draughts, the mean temperature at any point is constant, varying practically within the same limits for each cycle. The limits of temperature, however, and the times at which they occur, vary from point to point along the bar. Now, since the temperature variation at a given point depends only on the time, its relation may be written in the form—

$$\theta = A_0 + A_1 \sin\left(2\pi\frac{t}{T} + \delta_1\right) + A_2 \sin\left(4\pi\frac{t}{T} + \delta_2\right) + \text{etc.}$$

where θ denotes the temperature, t the time, reckoned from any arbitrary zero, T the periodic time, and the remaining symbols are constants. By means of a thermo-electric couple, such as that described in the previous article, the temperatures are noted for every minute of a complete cycle at some point near the centre of the bar, and from these observations the constants in the equation are determined. In a similar manner the terms of the equation—

$$\theta = A_0' + A_1' \sin\left(2\pi\frac{t}{T} + \delta_1'\right) + A_2' \sin\left(4\pi\frac{t}{T} + \delta_2'\right) + \text{etc.}$$

can be found for another point on the bar at some distance, l, from the first. As each set of observations must be made either simultaneously or with the same time relation to the heating and cooling, it is as well to start them when the steam is turned on for two successive cycles.

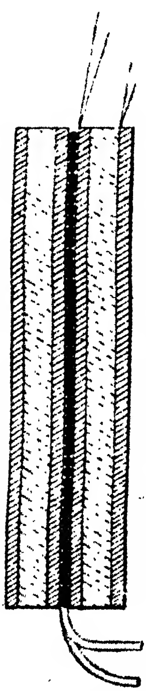
As the coefficients A_0, A_0', A_1, A_1', etc., rapidly diminish, it is only necessary to find the first three. This may be done algebraically, since the observations give twenty-four equations, from which but five unknown terms have to be found; or curves may be plotted, and the methods of Art. 56, Part I., may be employed.

These having been obtained, it may be proved that the diffusivity, k (*i.e.* the conductivity divided by the product of the specific heat and density), is given by the equations—

$$k = \frac{K}{\sigma d} = \frac{\pi l^2}{T(\delta_1 - \delta_1') \log_e\left(\frac{A_1}{A_1'}\right)} = \frac{2\pi l^2}{T(\delta_2 - \delta_2') \log_e\left(\frac{A_2}{A_2'}\right)} =, \text{etc.}$$

It is advisable, both as a test for the cyclic condition of the bar and for general accuracy, to take the time and temperature observations for each point twice, the sets of observations for the two points alternating, and so covering in all four successive cycles.

89.



To compare the Conductivities of Badly-conducting Substances by Means of Slabs.—A flat coil of insulated German-silver wire of known resistance is placed between two copper plates, to which thermo-electric couples are soldered. On either side of these is placed a thin slab of the substance to be investigated, and outside these two other metal plates with thermo-couples. To secure continuity of material, the faces of the metal plates against the substance may be covered with mercury or glycerine. The thermo-electric couples being standardized as explained in Art. 87, the temperatures of the inner and outer pairs of metal plates may be ascertained. If an electric current of C ampères be passed through the central coil, of resistance R ohms when hot, a quantity of heat equal to C^2R watts must travel outwards through the plates when a stationary condition of temperature is attained. Hence, if A be the area in square centimetres of each plate, T their thickness, θ_1, θ_2 the temperatures of the inner and outer metal plates respectively, and K the conductivity of the substance, since 1 watt = 0·240 calories per second -

$$\frac{0{\cdot}240C^2R}{2} = \frac{KA(\theta_1 - \theta_2)}{T}$$

$$K = \frac{0{\cdot}240C^2RT}{2A(\theta_1 - \theta_2)}$$

The value of K for different values of θ_1 and θ_2 should be determined by varying C, and results tabulated thus:

Material of slabs =
R of coil at temperature of air =
T =
A =

θ_1.	θ_2.	R at θ_1.	C.	C^2R.	K.

90.

To determine the Mean Temperature Coefficient for the Modulus of Rigidity of a Substance.—The investigation may be best made with a spiral spring made of the substance. This should be placed inside a tube having one end closed by a cork through which a wire by which the spring is suspended passes. Through this cork also passes a tube which may be connected to a boiling-can.

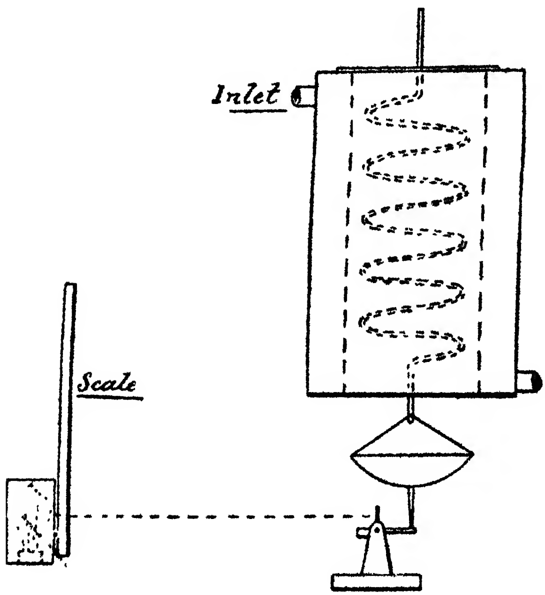
The lower end of the spring is attached to a straight wire carrying a pan, to the underside of which is soldered a pointed piece of stiff wire.

The pan is firstly loaded until the spring is carrying its maximum load but not extended outside its surrounding tube, and the vertical position of the point beneath the pan is observed by means of a cathetometer.

A small mirror fixed in a vertical plane to a horizontal axis, which also carries a projecting arm, is now adjusted in such a way that the pointed wire enters the small hole in the arm.

A lamp and scale are also adjusted, so that the reflected spot is at the scale's zero, and that its vertical displacements may be measured.

The temperature is noted, and steam is then blown through the tube, raising the spring's temperature to 100°. Any water trickling down the wire may be removed by touching it with blotting-paper. The position of the spot is now noted, and both mirror and scale removed. Steam is turned off, and the



spring allowed to fall to its original temperature, upon which the weights are removed from the pan, and the position of the point beneath it again observed by the cathetometer.

Let D be the difference of the cathetometer readings, *i.e.* the extension of the spring at the temperature T° of the air.

Let d be the alteration of extension when heated from T° to 100°, N the number of centimetres the spot moved over the scale, L the distance between the mirror and scale, l the length of the mirror arm from axis to the hole.

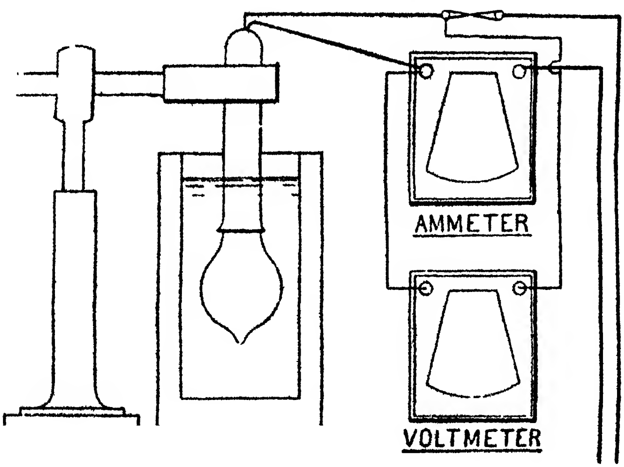
Then $\frac{d}{D(100^\circ - T)}$ is the temperature coefficient of rigidity, and $d = \frac{Nl}{2L}$ approximately, since the angle turned by the mirror is half that turned through by the reflected ray.

Therefore—

$$\text{Temperature coefficient of rigidity} = \frac{Nl}{2DL(100 - T)}$$

91.

To determine the Mechanical Equivalent of Heat electrically.—This equivalent, commonly known as "Joule's equivalent," is the ratio of the energy of a heat unit, or calorie, to that of one erg.



The method consists in producing heat by the expenditure of a known number of ergs electrically, absorbing this heat by water, and calculating its amount from the known weight of water and its rise of temperature.

The heat may conveniently be produced in an incandescent lamp immersed in the water.

Let the known water-equivalent of the calorimeter be WEq, and of the lamp w'.

Sufficient water is then poured into the calorimeter to completely immerse the lamp and about an inch of the holder, the latter being held in a clamp. This water is measured as poured in, and consequently its weight is known. Let it be W.

The lamp is then connected through an ammeter and switch to the mains, a voltmeter being also connected across it.

The temperature t of the water is noted, and current then switched on at a noted time. The voltmeter and ammeter are then read every half-minute, until the water, kept well stirred, has risen some 10 degrees. The time is again noted, and the current switched off. The highest temperature T attained by the water is then observed.

The observations should be tabulated thus:

Time.	Ampères.	Volts.	Ampères × volts × 30.

The sum of the entries in the last column is the total watt seconds; multiplying it by 10^7 the total number of ergs expended is found. The heat gained by the calorimeter, etc., is $(W + WEq + w')(T - t)$ calories. Hence, denoting the equivalent by J—

$$(W + WEq + w')(T - t) = \frac{\text{ergs expended}}{J}$$

$$J = \frac{\text{watt-seconds} \times 10^7}{(W + WEq + w')(T - t)}$$

The water-equivalent, w', of the lamp may be obtained with sufficient accuracy by assuming its entire weight to be of glass.

A preferable method of making this determination is to use a coil of insulated high-resistance wire instead of the lamp, the current being obtained at a lower voltage from a battery of cells.

The temperature of the water when current is switched on should be noted, as also at every succeeding half-minute until the temperature reaches about 60° C. The rate of cooling should then be determined, and curves plotted by the method of Art. 68. In this case T is given by the maximum ordinate of the derived curve.

The voltmeter may be dispensed with if the resistance R and the temperature coefficient of the resistance wire are known; as, denoting the coefficient by c, the watts per second at any temperature θ are given by $R(1 + c\theta)C^2$.

Results by this method should be tabulated thus:

W = WEq = R =

Time.	(θ) Temperature	Current.	Volts or $R(1 + c\theta)$.	Watts.	Watt seconds.

92.

To determine Temperature by Means of a Platinum Thermometer.—If the temperature of a length of platinum wire be altered, its electrical resistance also varies. Denoting the resistance at 0° C. by R_0, the temperature and resistance at T° are related according to the equation—

$$R_T = R_0(1 + aT + bT^2)$$

when a and b are constants. Further, if conditions could remain unaltered, the resistance at −273° C. would become zero, provided the metal be pure.

The platinum thermometer consists of a long fine platinum wire, supported on a mica frame, and enclosed in a glass or porcelain tube. With the handle are connected four terminals, two connected to the wire, the other two connected also to the wire, or to compensating leads, according as the resistance is

to be measured by the Wheatstone bridge, or by the Potentiometer method (Arts. 122, 120). The instrument is standardized, and the values of a and b are found, by firstly surrounding it for some time with melting ice at 0° C. and measuring its resistance R_0. The same operation is then performed after it has been raised to 100° C. by steam. Then—

$$R_{100} = R_0(1 + 100a + 10{,}000b)$$
$$\text{and } R_{-273} = R_0(1 - 273a + 74{,}529b) = 0$$

These two equations are, of course, sufficient to determine the two unknown quantities a and b. A curve may then be set out with various values of R as abscissæ, and the corresponding values of T as ordinates. The temperature of any enclosure into which the thermometer is placed may then be found by measuring the resistance and referring to the curve.

The assumption of zero resistance at $-273°$ may be avoided by finding the resistance for some third known temperature, for instance, 300° C. For the last equation may then be substituted—

$$R_{300} = R_0(1 + 300a + 90{,}000b)$$

PART III

MAGNETISM AND ELECTRICITY

MAGNETISM

THE strength of a magnetic pole is the force (in dynes) with which it repels a unit positive magnetic pole 1 cm. distant from itself. It will here be denoted by m.

The moment of a magnet is its pole strength multiplied by the length between its poles. It will here be denoted by M.

The intensity of magnetization is the magnetic moment of unit volume. It equals M divided by the volume of the magnet, or m divided by the sectional area, in the case of a bar. It is usually denoted by I.

93.

To map the Field due to a Given Distribution of Magnetism.—A small compass-needle should be placed in any part of the region to be mapped, and dots made with a sharp pencil, marking the position of its ends. It is then moved through its own length so that one end coincides with the mark which was previously under the other end, and the position of the leading pole is again marked. In this manner a series of points is found through which a continuous line is drawn. Similar lines are then plotted over the whole region. One or more positions are usually found in which the needle rests in any direction, owing to the weakness of the directive force. These are termed "neutral" points.

In plotting the field of a bar magnet, it should be placed in the magnetic meridian. Owing to the actual field plotted being the resultant of that of the magnet and that of the earth, the lines are not symmetrical at each end.

The equipotential lines cut the magnetic lines at right angles. They are most conveniently set out by means of a compass-needle carrying a pointer perpendicular to itself. The position of this pointer's ends is then plotted, and the lines drawn.

94.

To compare the Moments of Magnets by the Magnetometer.—A magnetometer consists of a suspended magnetic needle beneath which is a circle divided into degrees. In the more sensitive forms the indications are given by a mirror and scale.

The magnetometer is adjusted until the needle is in the magnetic meridian, and giving zero reading on the scale. The magnets to be compared are placed due east and west of the needle's centre, either on a line previously drawn, or along a bar permanently attached to the instrument. Similar poles of each are toward the needle, and their distances from it are adjusted until the needle lies in its original zero position. Let D_1 and D_2 be the distances between the centres of the magnets and the needle, M_1 and M_2 the moments of the magnets. Then, if D_1 and D_2 be large compared with the magnet's length—

$$\frac{M_1}{M_2} = \left(\frac{D_1}{D_2}\right)^3$$

The ratio should be found for several values of D_1 and D_2, both with north and south poles, the mean value being ultimately obtained.

In some cases it is difficult to measure to the centre of the needle, and this operation may be avoided by finding distances D_1 and D_2 at which balance exists, and increasing

them to D_3 and D_4 respectively, the balance being maintained. Then—

$$\frac{M_1}{M_2} = \left(\frac{D_3 - D_1}{D_4 - D_2}\right)^3$$

The relation between magnetic force and the distance from the magnet may also be investigated by the magnetometer. The magnet being placed due east or west of the needle, the tangent of the latter's deflection is proportional to the magnetic force due to the magnet at the needle. A curve should be set out having distances between the magnet and needle as abscissæ and values of the tangents as ordinates.

95.

To compare the Moments of Two Magnets by Oscillation.—When a magnet is suspended about its centre of mass and vibrating through a small arc in a uniform magnetic field, the force acting upon it is proportional to its displacement, and it consequently oscillates with a simple harmonic motion. Its period T is, therefore, equal to $2\pi\sqrt{\frac{1}{\text{acceleration}}}$.

But the acceleration is the twisting couple, or torque, divided by the moment of inertia I.

Hence, if H be the strength of field in which it swings—

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

$$\text{and } M = \frac{4\pi^2 I}{HT^2}$$

If two magnets be separately oscillated in the same uniform field—

$$\frac{M_1}{M_2} = \left(\frac{T_2}{T_1}\right)^2$$

while if H, the strength of field, be known, the moment of each may be absolutely determined when I is calculable.

The magnets should be carried during vibration in a wire stirrup attached to a thread as long as convenient. To avoid original torsion, a non-magnetic bar similar to the magnet should be suspended, and the support of the thread turned until it lies in the magnetic meridian. A thin straw or other suitable object should then be affixed to one end of the magnet, so that when the latter is suspended and at rest, an ink mark may be made on the glass cover in line with the straw and the suspending thread.

Care should be taken in suspending that the correct pole is toward the north, and that the magnet lies in a horizontal plane. Oscillation is started, after the mark on the glass has been made, by bringing another magnet near from due east or west of the centre and in the same horizontal plane. The period is determined in the usual manner (Art. 27).

$$\left.\begin{array}{l}\text{I for a round bar oscillating}\\ \text{about its centre of length}\end{array}\right\} = \text{mass}\left(\frac{\text{length}^2}{12} + \frac{\text{radius}^2}{4}\right)$$

$$\left.\begin{array}{l}\text{I for a rectangular bar of hori-}\\ \text{zontal dimensions } b \text{ and } l\end{array}\right\} = \text{mass}\left(\frac{l^2 + b^2}{12}\right)$$

This method may obviously be applied to compare the strengths of uniform fields, or, using a sufficiently small magnet, the relative strengths at different parts of one that is otherwise.

For, oscillating the same magnet in fields of strength H_1 and H_2, if N_1 and N_2 be the vibrations per second—

$$\frac{H_1}{H_2} = \left(\frac{N_1}{N_2}\right)^2$$

For this latter purpose a short massive magnet, suspended in a glass tube about 2 cms. diameter and 10 cms. long, is convenient.

Employing this method, the intensity all over a horizontal plane in some artificial field should be found, the direction being firstly plotted as in Art. 93.

For each of the lines drawn on the plane a curve may be plotted having the lengths of the line from some limit, set out to full size, as abscissæ, and the intensities at the corresponding points as ordinates. These intensities may be expressed relatively to the horizontal component of the earth's field as unity, the latter being found by vibrating in some place remote from iron objects.

If these curves be cut out and erected vertically along the lines to which they correspond, a solid diagram of both direction and intensity is obtained.

96.

To determine the Horizontal Intensity, H, of the Earth's Field.—The magnetometer of Art. 94 is placed in the position there described, *i.e.* with the index at zero.

A magnet is then placed at a distance D due east or west of the needle's centre, the measurement being from the centre of the magnet to that of the needle, and D being large compared with the magnet's length. As may be proved experimentally when M and H are known, if θ be the magnetometer deflection—

$$\frac{M}{H} = \frac{D^3}{2} \tan \theta$$

If now the magnet be oscillated in the earth's field, as explained in the last article—

$$MH = \frac{4\pi^2 I}{T^2}$$

where T is the period of oscillation, and I is the magnet's moment of inertia. From these equations—

$$H = \left(\frac{8\pi^2 I}{T^2 D^3} \cot \theta\right)^{\frac{1}{2}} = 2N\pi\left(\frac{2I \cot \theta}{D^3}\right)^{\frac{1}{2}}$$

if N denote the number of oscillations per second.

In determining θ, each pole in succession should be turned toward the needle, and the mean value of the deflections taken, each being regarded as positive.

97.

To determine the Magnetic Inclination or Dip; and hence, H being known, the Total Intensity of the Earth's Field.—A simple pattern of dip-circle consists of a flat magnetized needle having an axis at its centre and its ends pointed. It is supported on knife-edges, so as to be movable in a vertical plane. Coaxial with the needle is a graduated circle rigidly attached to the frame carrying the knife-edges. The latter should be capable of rotation about a vertical axis, its changes of azimuth being read from a graduated circle on the base.

The dip is the angle the resultant of the earth's magnetic field makes with a horizontal plane. In determining it, the base of the dip-circle is levelled, and the frame turned so that the needle lies approximately at right angles to the magnetic meridian. The frame is then adjusted in azimuth until the upper end of the needle is at 90° on the circle, and the azimuth reading is noted on the horizontal circle. The same observation is repeated with the lower end of the needle at 90°.

The needle is now turned so that its previously outer side is next the circle, and the same operations repeated.

After this, the whole frame is turned through 180° in azimuth, and an exactly similar set of observations made. The mean of these eight readings is the true easterly and westerly azimuth position of the dip-circle.

The frame is rotated through 90° from this mean position, and then lies in the magnetic meridian. The readings of both ends of the needle are noted. It is then turned so that the previously outer side is next to the circle, and the position of each end again read.

Exactly the same sequence of observations is again made with the frame turned through 180°.

The polarity of the needle should now be reversed by careful stroking with bar magnets, the poles of the latter being drawn according to the method of "divided touch" from the centre to the ends of the needle.

The last eight observations are then repeated, so that, in all, sixteen readings are obtained.

The mean of these gives the angle of dip, corrected for—

(*a*) Non-coincidence of centres of needle and circle.

(*b*) Non-coincidence of magnetic and geometrical axes of needle.

(*c*) Centres of mass and oscillation not lying upon the magnetic axis.

(*d*) The top and bottom scale readings of 90° not being situated upon a vertical line.

(*e*) Centre of mass not lying upon a line through the centre of oscillation at right angles to magnetic axis of the needle.

Results should be tabulated thus:

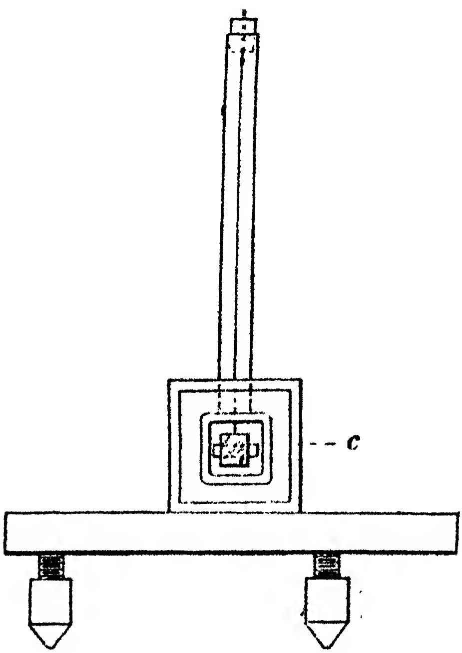
	Marked pole of needle.	
	North.	South.
Upper end		
Lower end		
Upper end, needle turned		
Lower end, „		
Upper end, frame turned		
Lower end, „		
Upper end, „ needle turned		
Lower end, „ „		

Mean angle of dip =

The total intensity equals H multiplied by the secant of the angle of dip.

98.

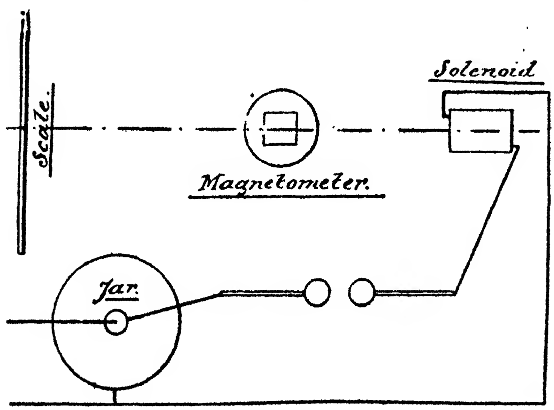
To determine by Means of a Magnetometer the Intensity of Magnetization produced in Steel Needles by Leyden Jar Discharges of Various Spark-lengths passing through a Given Solenoid surrounding them.—The magnetometer for this purpose is preferably a mirror instrument. One may be simply made by attaching a thin piece of mirror glass to



the side of a very small magnetic needle and suspending by a silk fibre. The case may be a small wooden block with a hole cut out as shown. A strip of copper or brass, *c*, should be bent round to fit this hole, soldered together, and drilled so that the suspension may pass through. A glass tube vertically fixed to the wood carries the fibre by means of a pin passing through

a cork at the top. The whole thing should be mounted upon a base with three levelling-screws.

It should be placed in a position remote from magnetic material, and a scale then adjusted in the magnetic meridian until the spot reflected from a lamp below the centre falls upon the zero of the scale. If a magnet of moment M be placed



due east or west of the magnetometer, the distance between their centres being D,

$$M = \frac{HD^3 l}{4L}$$

where L is the distance between the mirror and scale, l the permanent displacement of the spot over the latter.

The solenoid should be placed inside a glass tube, passing out through corks at the ends. This tube should be filled with oil, and the needle which is to be magnetized stuck centrally in one of the corks. A pair of brass balls on rods are supported by an ebonite frame in such a manner that their distance apart is adjustable. Connections should be made as shown. The wires from the Leyden jar are connected to the terminals of a Wimshurst or other electrical machine. A needle being in the solenoid, the balls are adjusted at some noted distance apart, and the machine turned until a spark passes.

The permanent displacement of the spot and D are observed, and the moment of the magnetized needle calculated.

The intensity of magnetization is then found by dividing the magnetic moment by the needle's volume, after which the observations are repeated on another for some new length of spark.

Results should be tabulated thus:

Spark gap.	D.	L.	Magnetic moment.

It will be noted that if D and L are constant, the moments are proportional to l, to which I also is proportional if all the needles have the same volume.

A curve should be plotted with lengths of spark as abscissæ and values of I as ordinates.

99.

To determine the Distribution of Magnetism along a Bar Magnet.—(*a*) *The Vibration Method.*—A short thick bar is highly magnetized and suspended by silk fibres both above and below, so that it may oscillate but cannot swing as a whole. These fibres may be secured to pins passing through corks in the ends of a piece of glass tube about 1·5 cm. diameter. This vibration magnetometer is clamped vertically, and the frequency of its oscillations per second found when in the earth's field alone. Let this be n. The bar magnet is then placed due north or south of its centre, vertical, and with its edge at a known distance from the tube. Starting with one end of the magnet in the same horizontal plane as the centre of the needle, the frequency is found. Let this be N_1. Raising the magnet by 1 cm., the frequency is again observed. Let it be N_2. The distance the end of the magnet is above the needle is better measured directly for each observation, and in the above-mentioned manner the vibration frequency, N_1, N_2,

N_3, etc., for noted positions along the magnet are found. This process should continue till the lower end is level with the needle, the attracted pole of which must always be nearest the magnet.

Then, if M_1, M_2, M_3, etc., denote the free magnetism at points corresponding to frequencies N_1, N_2, N_3, etc.—

$$\frac{M_2}{M_1} = \frac{N_2^2 - n^2}{N_1^2 - n^2}$$

$$\frac{M_3}{M_1} = \frac{N_3^2 - n^2}{N_1^2 - n^2}$$

etc.

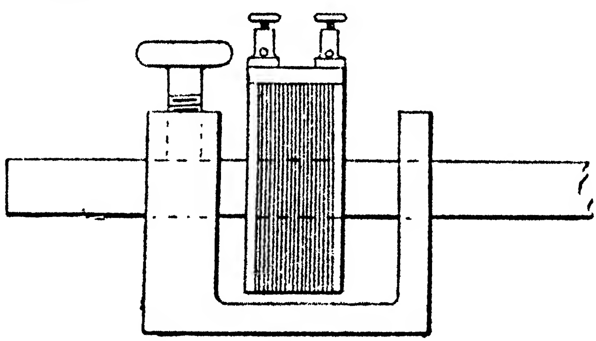
Results should be tabulated thus:

Distance from N end of magnet.	N.	N^2.	$N^2 - n^2$.

A line should be drawn equal in length to the magnet, and at the appropriate distances along it ordinates erected proportional to $N^2 - n^2$. The ordinates should be plotted below the line for that part of the magnet which attracts the north pole of the vibrating bar. The curve drawn through these points then shows the distribution of magnetism along the bar.

(*b*) *Rowland's Method.*—A coil wound on a bobbin which just slips over the magnet is here employed. This coil may be about a centimetre long, and is connected with a ballistic galvanometer. A jaw-piece of some non-magnetic material, having the gap between its arms 1 cm. wider than the over-all length of the coil, is capable of attachment to the magnet by a clamping-screw. This jaw is firstly fixed in such a position that one end of the magnet is flush with the inner side of one arm, the coil embracing that part of the magnet between the two. The galvanometer being now so distant as to be unaffected by the magnet, and the latter being held horizontally at right angles to the magnetic meridian, the coil is quickly slipped along from contact with the outer to contact

with the inner arm, and the ballistic throw noted. The position of the inner face of the arm nearest the magnet's centre is then marked, and the jaw-piece moved until the inner face of the other arm occupies the same position. The coil is slipped along, and the throw observed as before. These operations continue until the entire length of the magnet has been traversed. It has then been divided into a number of lengths equal to the width of the jaw, and the free magnetism, or number of magnetic lines issuing from each, is proportional to the ballistic throw obtained.



Results should be tabulated thus:

Width of jaw =

Distance from end of magnet to inside face of inner arm.	Throw.

A reflecting galvanometer should be used and the throw taken in scale divisions (see Art. 101).

As in method (*a*) a diagram should be made, in this case with values of the throw as ordinates, their abscissæ being the points midway between the two sides of the jaw-piece.

The ballistic throws are proportional to the electromotive

impulses in the coil, and these are equal to the number of turns in the coil multiplied by the magnetic lines cut across.

If the ballistic constant of the galvanometer and its resistance, also the number of turns in the coil and its resistance, are known, the absolute number of lines from any part of the magnet may be found.

For let Q be the quantity of electricity producing a throw of one division, l the throw observed, G the galvanometer resistance, R that of the coil, and N the number of turns in the latter; then—

$$lQ = \frac{\text{lines} \times N}{G + R}$$

$$\text{Magnetic lines cut} = \frac{lQ(G + R)}{N}$$

If Q is expressed in coulombs, G and R in ohms, it is necessary to multiply the right-hand term of the above expression by 10^8 to obtain the number of C.G.S. lines.

100.

To determine the Magnetic Force at Various Points along the Axis of a Current-carrying Coil.—This relation may be found by either the deflection or the oscillation method. In the first case, the axis of the coil is adjusted at right angles to the magnetic meridian, and the deflections of a needle read at various points from the centre of the coil outwards. The force at each due to the coil is then given by H tan θ.

For the oscillation method, the axis of the coil must lie in the meridian, and the current when switched on should pass round the coil clockwise if viewed from the south. Previously to switching on, the number of vibrations made per second by a short massive needle under the action of the earth's field should be noted in the usual manner. Let this be denoted by N. Starting at the centre of the coil, the number of swings

per second is found at various points with a constant current in the coil. Let these be N_1, N_2, etc. Then the force due to the coil at the various points is $\frac{N_1^2 - N^2}{N^2}H$, $\frac{N_2^2 - N^2}{N^2}H$, etc.

Results should be tabulated thus:

Distance from centre of coil.	Vibrations per second.	Force due to coil.

Oscillations per second in earth's field alone =

A curve should now be set out with the distances as abscissæ and the forces as ordinates.

Applying the method of Art. 94, with the coil substituted for one of the magnets, the other being of known moment, it may be shown that the magnetic moment of the coil is equal to the product of the number of turns, the mean area looped round by its turns, and the current in absolute units. (1 absolute unit = 10 ampères.)

101.

On Electrical Measurements and the Use of the Galvanometer.—Though not repeated for each determination, the following points require attention:—

In making connections, the ends of insulated wires should be stripped for about 2 cms. and scraped clean. The wires themselves should be as short as convenient. Their ends may be bent into a hook and put round the binding-screw, but should not be twisted round it.

Before any appliance is connected to points at different potentials, the possibility of an excessive current flowing through should be considered.

Secondary cells should never be short-circuited. The negative terminal may be identified, where the plates are visible, by its being connected to the two outside plates.

Care should be taken never to let wires from a main circuit be connected, except through a considerable resistance, still more never to let them make contact with each other. Neglect of this causes annoyance and delay in replacing fuses—or worse trouble.

The most common measuring instruments are the voltmeter, ammeter, and galvanometer. The two former are direct reading.

The galvanometer most generally used is the tangent instrument, reading in degrees from a pointer, or in divisions from a scale and mirror.

The suspended-coil type of galvanometer gives, when unifilar, deflections in which the angle itself is proportional to the current; but when bifilar, the tangents of the angles have this relation.

In any case, if the deflection of a mirror be small, the scale readings may be taken as approximately proportional to the current.

The "galvanometer constant," or "G," is such that—

$$\text{Current in ampères} = \frac{H}{G} \tan \theta \times 10$$

where H denotes the horizontal component of the earth's field, and θ is the angle of deflection. G really equals 2π times the number of turns in the coil divided by the mean radius.

It is useful to know the number by which the tangent may be multiplied to give the amount in ampères. As H is fairly constant for any one place, $\frac{10H}{G}$ is also approximately so for any particular position of the galvanometer. This quantity is termed the "reduction factor," and may be denoted by k.

In using an instrument reading from a pointer, the coil is turned until this is at zero. If there be several pairs of terminals, that pair should be used which gives a deflection nearest to 45°.

The reading of each end of the pointer is noted. The direction of the current through the coil is then reversed, and

the same readings again taken. The mean of the four should then be found, and its tangent ascertained from tables. The reading of each end corrects for any error in the relative position of pivot and scale. The reversal of current corrects for any between coil and scale.

With a mirror instrument the lamp should be under the zero of the scale, and both lamp and scale then adjusted vertically until the spot is on the graduated part of the latter. If the galvanometer be in adjustment, the spot should now be at zero; but should it not be, the position of the lamp may be altered slightly until it is.

Most of the determinations here dealt with, however, are by null methods, and in these the exact position of the spot is immaterial.

In many cases the "spot" is in reality a band, and one edge of it should then be read from.

If with a pointer instrument the pointer swings round to a permanent deflection of nearly 90°, the circuit must be immediately opened, as the current is probably excessive. With a mirror galvanometer only the smallest electromotive forces must be applied to the terminals.

A ballistic galvanometer is used to measure quantities of electricity discharged through its coil. It may be proved that—

$$Q = \frac{HT}{\pi G}\left(1 + \frac{\lambda}{2}\right)\sin\frac{\theta}{2} \times 10$$

where Q = quantity in coulombs, T = period of needle's vibration, λ = the logarithmic decrement, and θ = the ballistic throw, *i.e.* the amplitude of the first swing, H and G having their previous significance.

The term $\left(1 + \frac{\lambda}{2}\right)$ is introduced to correct for the damping, and λ may be found in the following manner. Let the needle be adjusted to zero, and, when at rest, started swinging over a considerable arc by bringing a magnet near for an instant. The reading at the end of a swing is noted, and that at the

end of the nth succeeding swing in the same direction. Let these be θ_1 and θ_2. Then—

$$\lambda = \frac{1}{2n}(\log_e \theta_1 - \log_e \theta_2)$$

The period of vibration, T, is found by noting with a stop-watch the time of a number of swings, and hence calculating that of one. It should be repeated several times, and the mean taken.

Methods of determining G and H are given in Arts. **104** and **96**. Where a mirror galvanometer is employed, and the throws are small, scale divisions may be taken as approximately proportional to quantities.

Proceeding as instructed, several currents should be measured, and the values of T and λ found for the galvanometer.

102.

To compare Electromotive Forces by the Galvanometer.—(*a*) *The Variable Deflection Method.*—A sensitive low-resistance galvanometer is connected through a large resistance with each source of E.M.F. in succession, and the deflections, θ_1 and θ_2, noted for each.

Let E_1, E_2, etc., be the E.M.F.'s; G, the galvanometer resistance; R, the large resistance put in; r_1, r_2, etc., the resistances in the remainder of each circuit.

$$\text{Then } \frac{E_1}{r_1 + R + G} \propto \tan \theta_1.$$

$$\frac{E_2}{r_2 + R + G} \propto \tan \theta_2$$

etc.

$$\text{and } \frac{E_2}{E_1} = \frac{r_2 + R + G}{r_1 + R + G} \cdot \frac{\tan \theta_2}{\tan \theta_1}$$

$$\frac{E_3}{E_1} = \frac{r_3 + R + G}{r_1 + R + G} \cdot \frac{\tan \theta_3}{\tan \theta_1}$$

etc.

If r_1, r_2, etc., and G are small compared with R—

$$\frac{E_2}{E_1} = \frac{\tan \theta_2}{\tan \theta_1}$$

$$\frac{E_3}{E_1} = \frac{\tan \theta_3}{\tan \theta_1},$$

etc. approximately

The internal resistances of cells and dynamos are small. R should obviously be as large as possible, subject to a clear deflection being given.

(*b*) *The Variable Resistance Method.*—In this case, the galvanometer being as above, a large resistance is put in each circuit and adjusted till the deflection is the same in both. Let R_1, R_2, etc., be these large resistances, other symbols being as before.

$$\text{Then } \frac{E_1}{r_1 + R_1 + G} = \frac{E_2}{r_2 + R_2 + G}, \text{ etc.}$$

$$\text{and } \frac{E_2}{E_1} = \frac{r_2 + R_2 + G}{r_1 + R_1 + G}$$

$$\frac{E_3}{E_1} = \frac{r_3 + R_3 + G}{r_1 + R_1 + G},$$

etc.

or, under the conditions mentioned above—

$$\frac{E_2}{E_1} = \frac{R_2}{R_1}$$

$$\frac{E_3}{E_1} = \frac{R_3}{R_1},$$

c. approximately

(*c*) *The Sum-and-difference Method.*—The electromotive forces to be compared are dealt with in pairs. Let two sources of E.M.F., E_1 and E_2, be coupled in series, so that both act in the same direction. The total E.M.F. is then $E_1 + E_2$. The deflection, when any convenient resistance is in circuit with them and the galvanometer, is noted. Let it be θ_1. One source, which may be taken as of E.M.F. equal to E_2, is

then connected in the opposite direction, so that its E.M.F. opposes that of value E_1. The total E.M.F. is now $E_1 - E_2$. Let this give a deflection θ_2, which may be positive or negative relatively to θ_1.

$$\text{Then } \frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

$$\text{and } \frac{E_1}{E_2} = \frac{\tan\theta_1 + \tan\theta_2}{\tan\theta_1 - \tan\theta_2}$$

Similarly with any other pair, E_1 and E_3.

(*d*) *Wheatstone's Method.*—One source of E.M.F. is connected to the galvanometer through a convenient resistance, and its deflection, θ_1, noted. A sufficient resistance, R_1, is then added until the deflection is brought down to a noted angle, θ_2. Let r_1 be the total original resistance before R was added.

$$\text{Then } \frac{E_1}{r_1} \propto \tan\theta_1$$

$$\frac{E_1}{r_1 + R_1} \propto \tan\theta_2$$

Repeating the operation with the other source of E.M.F., and adjusting till θ_1 and θ_2 are as before, let R_2 be the added, and r_2 the original, resistance.

$$\text{Then } \frac{E_2}{r_2} \propto \tan\theta_1$$

$$\frac{E_2}{r_2 + R_2} \propto \tan\theta_2$$

$$\text{Hence } \frac{E_1}{E_2} = \frac{r_1}{r_2}$$

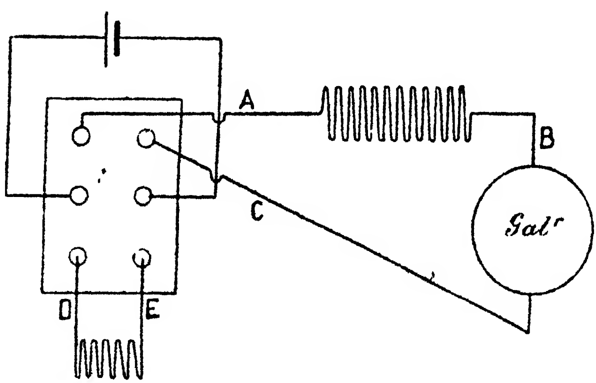
$$\text{and } \frac{E_1}{E_2} = \frac{r_1 + R_1}{r_2 + R_2}$$

$$\text{consequently } \frac{E_1}{E_2} = \frac{R_1}{R_2}$$

103.

To determine the Variation of Electromotive Force with Time for a Leclanché Cell with a Circuit of Given Resistance.—The cell should be connected to a switch, as shown, so that the current may pass round either the circuit ABC or DE as required.

DE is the resistance forming the external circuit through which the cell discharges, and may conveniently be several ohms. ABC contains considerable resistance, in series with which a galvanometer is connected. This instrument should



be as nearly dead beat as possible, and the resistances are so adjusted that a large deflection is given when an E.M.F. of 1·4 volt is applied to the circuit, and that their combined resistance is about 1000 ohms.

These resistances all remain unchanged during one set of observations. The latter commence by placing the contact-piece in the cups and on to A and C. The galvanometer then gives a deflection proportional to the E.M.F. at AC, which is practically the same as that of the cell on open circuit.

The current is then allowed to run for 30 seconds through DE, after which it is switched over to AC, and the deflection again noted. The time during which CD is switched out

should be as short as possible. This operation is continued until an apparently stationary deflection is given for several 30-second periods.

The circuit DE should then be removed, and the cell left on open circuit, the deflection when it is switched on to AC being noted every 30 seconds as before.

Results should be stated thus:

Closed circuit.

Time.	Deflection.

Open circuit.

A curve should be plotted with times as abscissæ and deflections as ordinates. It is convenient to plot the times of the open-circuit observations backwards from the last recorded time of the preceding set.

Similar observations should be made and curves drawn for different values of the resistance in DE.

104.

To determine the Electro-chemical Equivalent of Hydrogen, or, knowing this, to find the Reduction Factor of a Galvanometer.—The most convenient instrument for this purpose is a Hoffman's voltameter, consisting of two vertical glass tubes, fitted at the top with cocks and at the bottom with platinum electrodes. By opening the cocks and pouring into a third tube communicating with both, these tubes may be filled with dilute sulphuric acid, upon which the cocks are again closed.

A simpler apparatus, consisting of two platinum electrodes immersed in a vessel of dilute acid, and a glass vessel or tube, filled with the same, placed over one of them, will, however, suffice.

The electrolyte should contain about 1 part of strong acid to 10 of water.

That electrode from which the hydrogen is to be collected is now connected to the negative pole of a battery of several secondary cells, or through a lamp to the leads of the lighting circuit. The other is connected through the galvanometer or ammeter, and a key, to the other terminal. It is as well to connect the galvanometer to a commutating switch, so that the direction in which the current flows through it may be reversed. The six-hole mercury switch before mentioned (Art. 103) serves both as commutator and key.

The time is noted and the current started, the galvanometer deflection being observed now and at each succeeding minute. When 15 cms. or so length of the tube are occupied by hydrogen, the current in the galvanometer should be reversed. This, however, cannot be done with most ammeters. Observations continue until about 30 cms. of the tube are filled, and then the current is cut off.

Let the volume per unit length of the tube, previously found if necessary, be v, and let n cms. length of the tube be occupied by the gas.

The difference of liquid level inside and outside the tube is noted. Let this be h.

Then, if the height of the barometer be H, the gas is under a pressure $H \pm \frac{h}{13 \cdot 6}$ cms. of mercury, approximately, the positive sign being taken with Hoffman's voltameter, the negative with the other form described.

The total pressure of the gas in the tube, however, is that due to the pressure of the hydrogen, plus the maximum vapour-pressure of water at the existing temperature.

The latter being noted, the vapour-pressure may be found from tables.[1] Let it be p, and the temperature T.

Then the volume of hydrogen at 0° and 760 mms. is—

$$nv \times \frac{273}{273 + T} \times \frac{H \pm \frac{h}{13\cdot5} - p}{760}$$

and, since 1 c. cm. at 0° and 760 mms. weighs 0·0000896 gram—

$$\text{Weight of hydrogen} = 0\cdot0000896nv\frac{273(H \pm \frac{h}{13\cdot5} - p)}{(273 + T)760}$$

The electro-chemical equivalent is the weight in grams liberated by the passage of 1 coulomb of electricity.

Results of observations should be tabulated thus:

Time.	Current in ampères.

Let the mean current, multiplied by the time in seconds, be M coulombs. Then—

$$\text{Electro-chemical equivalent of H} = \frac{\text{weight of hydrogen}}{M}$$

If the electro-chemical equivalent be known, and the reduction factor of the galvanometer is required, values of the deflection are tabulated in the second column. Let the mean deflection be θ. Then, if k denote the reduction factor—

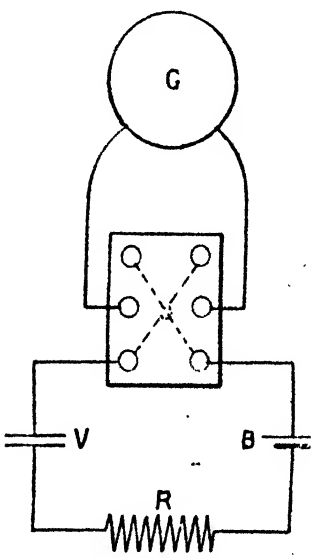
$$\left.\begin{array}{l}\text{Mean current in}\\ \text{ampères}\end{array}\right\} = k \tan \theta$$

$$\left.\begin{array}{l}\text{But mean current}\\ \text{in ampères}\end{array}\right\} = \frac{\text{total weight of hydrogen}}{\text{total time} \times \text{electro-chemical equivalent}}$$

Hence k may be determined.

105.

To determine the Electro-chemical Equivalent of Copper. —In this determination a current from one or two cells of constant E.M.F. is passed through a galvanometer or ammeter, G, and a copper voltameter, V. The latter consists of copper electrodes immersed in a solution of copper sulphate. It is advantageous for the anode to be of a U shape, with the kathode plate between its sides. The copper sulphate solution should not be saturated, but of any density less than 1·18, and it should be acidified with a few drops of sulphuric acid.



A commutating switch for the galvanometer may also be included in the circuit, and an adjustable resistance.

The circuit should be connected, and the resistance varied until a deflection of about 45° is produced.

The kathode is then removed, thoroughly scoured with glass paper, washed successively in potash or sodium hydrate solution, diluted nitric acid, and distilled water. It may then be dried by dipping in spirit or ether, and warming.

It is now, without being touched by the fingers, weighed with the greatest possible accuracy, and replaced in the voltameter.

The current is then switched on, and the time simultaneously noted. Observations of the current or deflection are made every minute for about 30 minutes, and entered thus:

Time.	Current in ampères.

At the middle of this period the current through the galvanometer should be reversed.

The kathode is then removed, quickly washed in distilled water, dried as before, and again weighed.

Let the increase of weight be W, and the coulombs of electricity which passed, *i.e.* the mean current multiplied by the time of flow in seconds, be M.

Then the electro-chemical equivalent equals—

$$\frac{W}{M} \text{ grams}$$

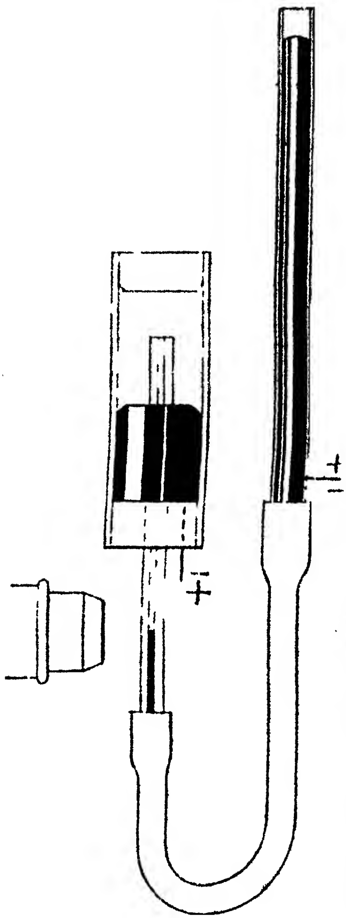
If the electro-chemical equivalent be known, the reduction factor may be determined for the galvanometer, as explained at the end of the last article.

The density of the current in the voltameter must not exceed $\frac{1}{50}$ of an ampère per square centimetre surface of the kathode.

106.

To construct and calibrate a Capillary Electrometer.—A short piece of glass tubing is closed at one end by a cork, and through the latter is passed a length of fine-bore (0·5 to 1 mm.) thermometer tube, previously washed through with nitric acid and distilled water in succession. The lower end of this tube is connected by rubber pipe with a wider tube, of which the height may be adjusted. The terminals are connected to two platinum wires, one passing through the cork into the wide glass tube, the other fused into the adjustable pressure tube near its lower end. Pure mercury is poured into the latter tube until it flows out from the upper end of the capillary, and a further quantity is poured into the

corked tube, covering the cork to a depth of 2 or 3 cms.



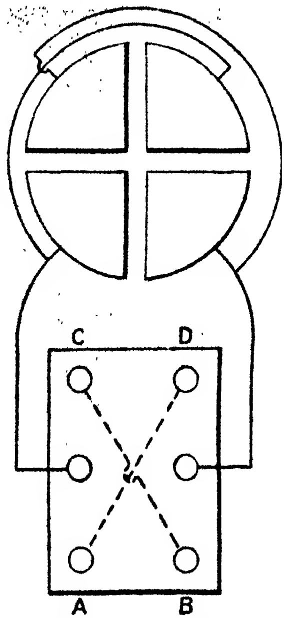
Dilute sulphuric acid, about 20 per cent. in strength, is now poured over the mercury, completely immersing the top of the capillary tube. It only remains to lower the pressure tube until the acid enters the capillary, and to adjust the meniscus in a convenient position.

This being done, a microscope cathetometer is focussed horizontally upon the meniscus, and the variation of its level for various small potential differences at the terminals is observed. These should be of the order 0·001 or less of a volt, and may conveniently be obtained by tapping points along a calibrated wire with a small voltage across its ends. It is important to note that the terminals of the electrometer must be short-circuited after each observation. A table should be made out giving corresponding values of potential difference and displacement, the difference in level between the mercury in the capillary and the pressure tube when uncharged, and the temperature.

Electromotive force.	Displacement.

107.

To compare Electromotive Forces by Means of the Quadrant Electrometer.—For the comparison of electromotive forces of a few volts, the instrument should be used heterostatically, and with the needle charged to a high potential. For details of the method of maintaining the needle at this potential, the pamphlet issued with the instrument should be consulted.



The quadrants of the electrometer are connected to two mercury cups in a highly insulating ebonite or paraffin block. These two cups are then connected by a stout wire, and the spot adjusted to the zero of the scale. The wire connecting the cups is removed, and the rocking arm inserted.

The cell of electromotive force V_1 now has its terminals connected to A and B. The deflections are noted with the quadrants connected successively to A and B, C and D. Let the mean of these be d_1. The same connection is now made for the cell of electromotive force V_2. Let the mean deflection be d_2. Then—

$$\frac{V_1}{V_2} = \frac{d_1}{d_2}$$

If the replenisher is not used, and the potential of the needle gradually falls during the observations, similar poles of the two cells may be simultaneously connected to A, and the other pole of each connected successively to B. Let the

deflections be d_1 and d_2, with the quadrants connected to A and B. The same observations are, then made with the quadrants on C and D. Let the deflections, with the order of succession reversed for the cells, be d_3 and d_4. Then, although the potential of the needle may have altered slightly—

$$\frac{V_1}{V_2} = \frac{d_1 + d_4}{d_2 + d_3} \text{ approximately}$$

If the electromotive forces to be compared are large, the needle may be, with far greater convenience, connected to one pole of a water battery, or of several secondary cells in series, the other pole of which is earthed.

In this case also the electrometer may be used idiostatically, *i.e.* with the needle connected to one pair of the quadrants. The electromotive forces are then proportional to the square roots of the deflections.

108.

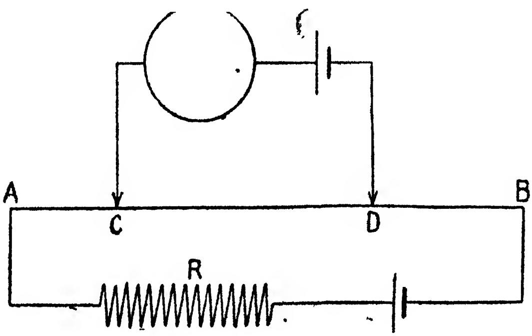
To determine Electromotive Forces by Means of the Potentiometer.—A potentiometer essentially consists of a long wire, AB, of uniform resistance per unit length, the ends of which are maintained at some constant known difference of potential. This condition is usually attained by connecting them through an adjustable resistance, R, to a secondary cell.

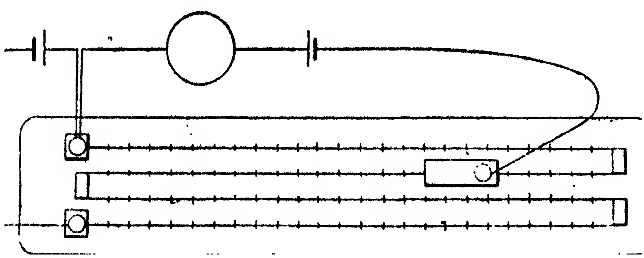
The length AB is usually graduated. Let it be N divisions long. Let the potential difference between A and B be V volts. Then, if a shunt circuit containing a galvanometer be connected to any two points on AB, at a distance apart equal to n divisions, a deflection is given by the shunt current through it.

Since the fall of potential from A to B is uniform, this current is due to an E.M.F. equal to $\frac{n}{N}V$ between C and D, acting round the circuit CEDC. If, therefore, a source of E.M.F., equal and in the opposite direction, were placed in CED, no current would pass through the galvanometer.

Hence, to determine the E.M.F. of a cell, which must, however, be less than V, it is placed in such a shunt circuit and the distance CD found, for which there is no deflection. Then the E.M.F. of the cell equals $\frac{n}{N}$V.

If the E.M.F. to be determined be greater than that producing the difference of potential V, the resistance R may





be removed, the secondary cell put in the circuit CED, and the unknown E.M.F. connected to A and B. Since the secondary cell has an E.M.F. of 2 volts—

$$\text{E.M.F. between A and B} = \frac{2N}{n}$$

Precaution must be taken that the wire is not overheated by the current.

It is obvious that the sensitiveness of a potentiometer is

proportional to the length of its wire. For convenience, the latter is usually made in short lengths connected by massive brass pieces. In practice also the point C is fixed, and coincides with A.

To determine an electromotive force, a Clark cell is firstly placed in CED, and balance found. The resistance R may be inserted in series with the potentiometer wire, and adjusted until n is some convenient number. Now, the E.M.F. of a Clark cell is $1{\cdot}435\{1 - 0{\cdot}00077(t^\circ - 15^\circ)\}$, where t is the existing temperature. Hence the drop of potential along AB is $\frac{1{\cdot}435}{n}$ volts per division at 15°, and nearly so at all ordinary temperatures. The resistance R remaining unaltered, any other source of E.M.F. may be substituted for the Clark cell, and its magnitude found by noting the number of divisions giving a balance.

109.

To determine the Electromotive Force necessary to produce Sparks across Air-gaps of Various Lengths.—The two sparking terminals may conveniently be spherical, and mounted as described in Art. 98. The electromotive force is produced by means of a Wimshurst or other induction machine, and measured by a simple form of attracted disc electrometer. The latter consists of two circular metal discs, the upper of which is supplied with a guard-ring and with threads, by means of which it may be suspended from the beam of a balance.

The distance, l, between the inner faces of the lower disc and the guard-ring is measured, also the diameter of the upper disc, and hence its area, A, is found.

The disc is now suspended from the balance and counterpoised, after which a small additional weight, W, is placed in the pan, thus raising the disc, to which the remaining portion of the electrometer is then adjusted in such a way that the guard-ring and disc lie in the same plane.

One pole of the machine is connected to the upper disc by

a fine wire just in contact, also to the guard-ring, and to one sparking terminal; the other pole to the lower plate and the other sparking terminal. The two latter, being at first in contact, are gradually drawn apart while the machine is in action, until the suspended plate is drawn down, when their distance apart, L, is observed.

The weight W is varied, several observations being made for each value, and the mean taken.

Now, if v be the electrostatic potential difference when the plate is drawn down—

$$v = l\sqrt{\frac{8\pi \times 981W}{A}}$$

consequently the electro-magnetic electromotive force in volts, V, is given by

$$V = l\sqrt{\frac{8\pi \times 981W}{A}} \times 300$$

A curve should be plotted having values of V as abscissæ and of L as ordinates.

Resistance.

The specific resistance of a substance is the resistance of unit length of a bar having unit sectional area.

The reciprocal of this is termed "the specific conductivity."

These are independent of the current in the bar, but vary with changes of temperature and strain.

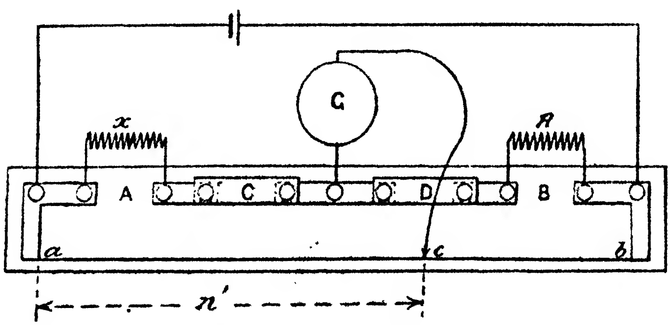
The practical unit of resistance is the ohm, which equals 10^9 absolute units.

The most generally used method of determining resistances is to compare them with some standard by means of a Wheatstone's bridge.

In practice this may take either of two forms: (*a*) the metre bridge; (*b*) the post-office resistance box.

110.

To determine a Resistance by the Metre Bridge.—The bridge consists of a wire, *ab*, which is commonly assumed to be of uniform resistance per unit length, but which should, for work of any accuracy, be calibrated, so that the resistance of any part of it is known. This is stretched between copper blocks as shown. These blocks form a bar interrupted by four gaps, on each side of which are screws. The cell is connected to the ends, and the galvanometer from the centre to a contact



piece, *c*, movable along *ab*. One gap on each side, suppose C and D, is closed by a short bar screwed across; in one of the others, suppose B, a standard resistance R is inserted; and in the other, A, the resistance x to be determined.

The contact-piece is now moved along *ab* until upon making contact no deflection occurs. The distance n' is noted. The resistances in A and B are now interchanged, and the distance n'' from *b* at which balance exists is observed.

Let the mean of n' and n'' be n. Then, if, as is usual, *ab* is divided into 100 parts,

$$\therefore \quad x = \frac{n}{100 - n} R$$

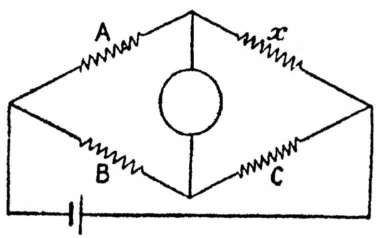
If the resistances of lengths along *ab* be known, it is

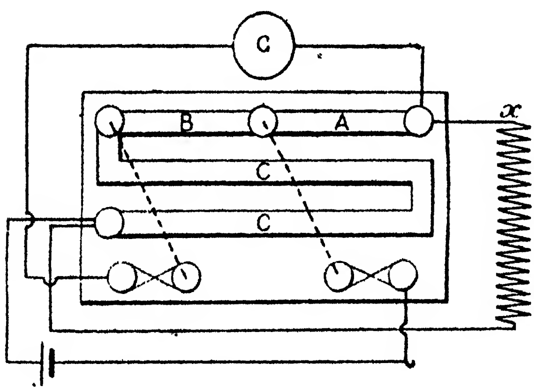
unnecessary to interchange x and R, and from the first observation—

$$x = \frac{\text{resistance between } a \text{ and } c}{\text{resistance between } b \text{ and } c} R$$

111.

To determine a Resistance by Means of a Post-office Resistance Box.—In the post-office box the three arms, A, B, and C, of the bridge are arranged as shown, and one





connection of both battery and galvanometer is made through the contact-keys in front.

In some boxes the relative positions of galvanometer and battery are interchanged; but, as the keys and the arms are usually indicated on the top of the box, no difficulty arises.

All screws should be well screwed down, and the plugs well in originally. The 10-ohm plugs should then be withdrawn from arms A and B, and firstly the battery key, and then the galvanometer key, depressed. The contact of the latter should be but momentary, and the direction of deflection noted.

The infinity plug is withdrawn and the deflection given upon depressing the keys again observed. If all connections and contacts are as they should be, these two deflections are in opposite directions. Assuming this to be the case, the resistance of arm C is varied by pulling out plugs until the deflection is in one direction or the other, according as the 1-ohm plug is in or out. If the other plugs withdrawn from C when this condition is obtained amount to N ohms, the resistance of x lies between N and N + 1 ohms.

Should the resistance of x be greater than the total value of all the plugs in arm C, the resistance in A must be increased from 10 to 100 or 1000 ohms. In this case the resistance in C which gives a balance is one-tenth or one-hundredth respectively of that of x.

On the other hand, where the measured resistance is small, that of A should remain 10 ohms, and the 100 or 1000 ohm plug, instead of the 10 ohms, should be drawn from B. The resistance in C which gives a balance is then 10 or 100 times that of x.

In determining large resistances, it is necessary to use a battery of several cells, and particular care must then be taken that one plug is always out of arm B while the key is depressed. Otherwise one of the smaller resistances in arm C may be burnt out.

112.

To determine the Specific Resistance of a Substance.—Since the specific resistance is the resistance of 1 cm. length of a conductor of 1 sq. cm. sectional area, the resistance of a length, L, of a circular wire of diameter D is—

$$R = \text{specific resistance} \times \frac{4L}{\pi D^2}$$

or, if R be known—

$$\text{Specific resistance} = \frac{\pi D^2 R}{4L}$$

It may be determined, therefore, by finding the resistance of a wire of measured length and diameter. The latter may be found by applying a micrometer gauge at several points, and taking the mean reading.

As some uncertainty with regard to the length is involved by the fact of the ends of the wire being connected to binding-screws, the best method of procedure is to find the resistance of about a metre length, marking on one end the position of the edge of the screw-head; then to slide the wire past the screw until about half its length lies between the points of connection, and to again mark the position of the screw-head and find the resistance.

The difference between the two values of the resistance is that of a length equal to that between the marks.

113.

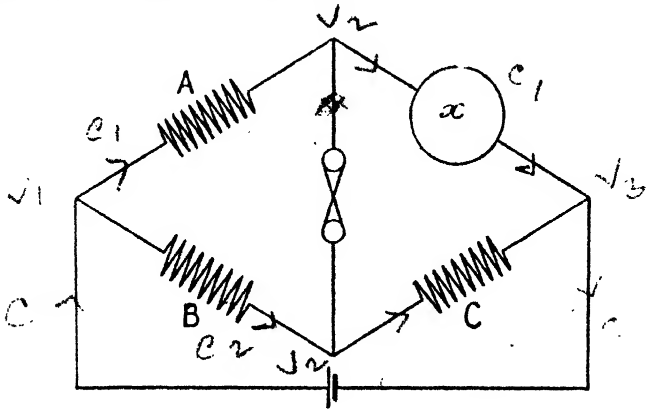
To determine the Resistance of a Galvanometer or Ammeter bv its own Deflection.—(*a*) *Thomson's Method.*—The galvanometer is put in one arm of a bridge, having the resistances of its other arms considerable. In the place usually occupied by it is put a key, and if the instrument be at all sensitive the battery should be shunted.

When A, B, and C are so adjusted that $\frac{A}{B} = \frac{x}{C}$, no current will pass through the key when it is depressed. Hence the deflection will be unaffected.

The battery is shunted and the resistances A, B, and C firstly adjusted until a convenient deflection is obtained, after which further adjustments are made until the deflection is constant, whether the key be open or closed. Then—

$$x = \frac{AC}{B} \text{ ohms}$$

It is convenient to insert an adjustable resistance in the battery circuit, so that the deflection may be adjusted without alteration of the ratios of the arms.



The galvanometer if sensitive may be shunted while in the bridge. Let S be the resistance of the shunt. Then—

$$x = \frac{ACS}{BS - AC}$$

(*b*) The resistance of a galvanometer may also be approximately found by connecting it in series with a post-office box, a key, and some primary cell of constant electromotive force. The cell should be short-circuited by a shunt of low resistance, and a considerable resistance put in circuit, before the key is depressed. The resistance is adjusted until a large deflection is given. This being R, and the galvanometer resistance G, the whole resistance of the circuit is R + G, if that of the cell and shunt be regarded as negligibly small. Let resistance now be put in until the deflection is halved, and let the whole resistance in circuit then be R′ + G. In this case—

$$2(R + G) = R' + G$$
$$G = R' - 2R$$

In a similar manner, if the galvanometer be shunted so

that the combined resistance is negligible, the resistance of the cell may be found.

The above remarks refer to a reflecting instrument. If it be the index form, and θ, θ' be the two deflections, one of which is not necessarily double the other, then—

$$(R + G) \tan \theta = (R' + G) \tan \theta'$$

$$G = \frac{R' \tan \theta' - R \tan \theta}{\tan \theta - \tan \theta'}$$

The resistance of a voltmeter may be found by this last method. It should be firstly put directly across the cell terminals, and then have the resistance put in circuit. In this case the resistance of the cell may be neglected. For an ammeter, method (*a*) is more suitable, all arms having their resistances small.

114.

To determine a High Resistance by the Direct Deflection Method.—The resistance is connected in series with a sensitive galvanometer and a battery. Let θ_1 be the deflection. A standard high resistance is now substituted for the other, and the new deflection θ_2 noted.

Then, if the E.M.F. of the battery has been constant, and R and x denote the standard and unknown resistances, G the resistance of the galvanometer—

$$\text{or } x = \frac{\theta_2}{\theta_1}(R + G) - G$$

If it is necessary to shunt the galvanometer, let S be the resistance of the shunt. In this case—

$$x = \frac{\theta_2}{\theta_1}\left(R + \frac{SG}{G+S}\right) - \frac{SG}{G+S}$$

The battery resistance, being comparatively small, has been neglected, if G also be small.

$$x = \frac{\theta_2}{\theta_1}\text{R approximately}$$

If the reduction factor k for the galvanometer, and the electromotive force V of the cell, be known, and θ be the deflection when the unknown resistance is in circuit with both—

$$k\theta = \frac{\text{V}}{x + \text{G}}, \text{ or } x = \frac{\text{V}}{k\theta} - \text{G}$$

and if shunted—

$$k\left(\frac{\text{G}+\text{S}}{\text{S}}\right) = \frac{}{x + \frac{\text{GS}}{\text{G}+\text{S}}}$$

or approximately—

$$x = \frac{\text{VS}}{k\theta(\text{G}+\text{S})}$$

115.

To determine Very High Resistances, such as those of Dielectrics, by the Electrometer.—Let a condenser of capacity K be short-circuited through a high resistance R, and let V be its potential at any instant.

Then the charge equals KV, and the current, *i.e.* the change of charge with time, is $-\text{K}\frac{d\text{V}}{dt}$. But by Ohm's law the current through the resistance equals $\frac{\text{V}}{\text{R}}$;

$$\text{therefore } -\text{K}\frac{d\text{V}}{dt} = \frac{\text{V}}{\text{R}}$$

$$\text{and } \frac{d\text{V}}{dt} = -\frac{1}{\text{KR}}\text{V}$$

The solution of this equation is—

$$V = A\epsilon^{-\frac{1}{KR}t}$$

$$\text{or } \log_\epsilon A - \log_\epsilon V = \frac{t}{KR}$$

where A is a constant determined by the initial potential.

Consequently, if the potential be V_1 at a time t_1, and at some later time t_2 it be V_2—

$$\log_\epsilon V_1 - \log_\epsilon V_2 = \frac{t_2 - t_1}{KR}$$

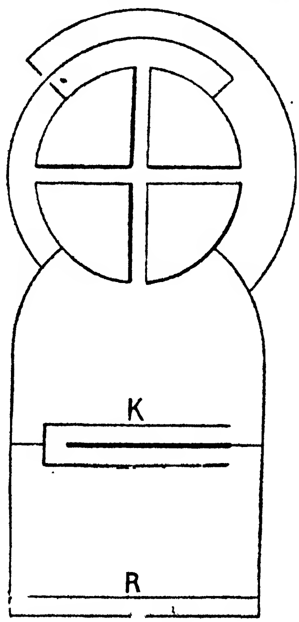
$$\text{or } R = \frac{t_2 - t_1}{K \log_\epsilon \frac{V_1}{V_2}} = \frac{t_2 - t_1}{2{\cdot}3K \log_{10} \frac{V_1}{V_2}} = \frac{t_2 - t_1}{2{\cdot}3K \log_{10} \frac{\theta_1}{\theta_2}}$$

where θ_1 and θ_2 are the deflections of a heterostatic electrometer.

A condenser of known capacity has its terminals connected to a quadrant electrometer, and is then charged to a potential which gives a considerable deflection. To effect this it is sufficient to touch its terminals momentarily with those from the battery.

The leakage of the condenser is now observed; if the deflection falls at all rapidly, the condenser resistance itself must be found as explained above. Let it be r.

The resistance is now put across the condenser terminals as shown, and the same observations made. If the combined resistance be R′—



$$\frac{1}{R'} = \frac{1}{R} + \frac{1}{r}$$

$$R = \frac{rR'}{R' - r}$$

The determination should be made for several different values of θ_1 and θ_2, with the condenser charged in opposite directions, and the specific resistance calculated. The greatest care is necessary to keep all connections as short as possible; the wires should not lie on anything, and the resistance should be suspended by silk threads.

116.

To determine Very Small Resistances by Means of a Post-office Box.—It is assumed that the unknown resistance is in the form of a straight bar, and that a standard resistance of comparable value is available. Let x be the unknown, R the standard resistance. Both are connected in series with a commutator and battery. The terminals of the standard are also connected to the ends of two arms of a post-office box as shown.

Contact with the unknown resistance may be made by a slotted piece of ebonite carrying one fixed terminal with a pointed end, and another similar, but capable of having its distance from the other adjusted by being moved in the slot. These two terminals are connected as shown to points on the arms.

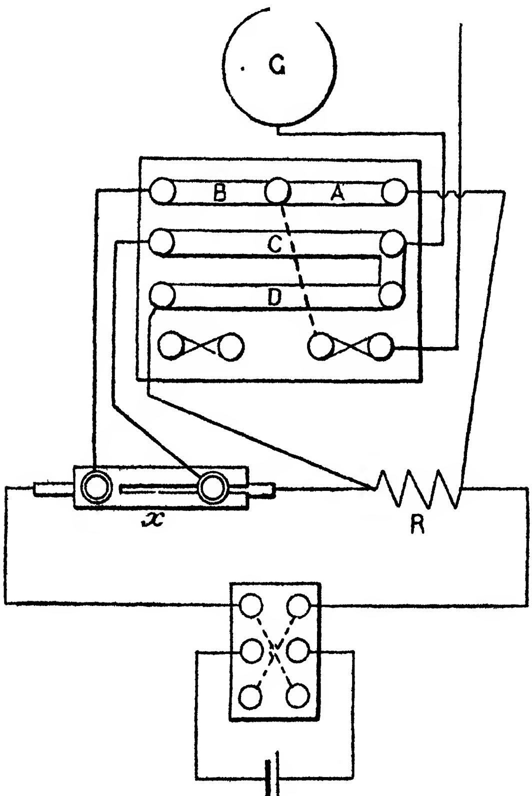
The bar between B and C is removed, plugs are withdrawn from A and B, and from C and D. The ebonite contact-bar is placed on the bar of unknown resistance, and the distance apart of its contact-points, together with the resistances in C and D, adjusted until no deflection occurs upon closing firstly the battery, and then the galvanometer circuit. The direction of current is then reversed, and balance again found. For each of these balances the distance apart of the contact-points is measured, C and D remaining unaltered when the current is reversed. The mean of these lengths should be taken as that for which the resistance is measured.

Now, the difference of potential between the contact-points is Cx, and that between the ends of the known resistance is

CR, if C be the current flowing through the battery. Therefore, when a balance exists—

$$\frac{Cx}{B+C} = \frac{CR}{A+D}$$

$$x = R\frac{B+C}{A+D}$$



Hence, if the bar be of uniform section, and the distance of the contact-points apart be L, its resistance per centimetre length is—

$$\frac{R}{L}\left(\frac{B+C}{A+D}\right)$$

It is advisable to open the battery circuit when observations are not actually being made.

117.

To determine the Internal Resistance of a Cell by Means of the Galvanometer.—This method is only applicable to cells of fairly constant electromotive force. A galvanometer, resistance box, and commutator are placed in series with the cell. If the latter be of low resistance, the galvanometer should also be so, and the resistance box should be capable of carrying a moderate current. Resistance should be put in circuit until the deflection is about 60°. Let this be R_1, and the deflection θ_1. It is then increased to a value R_2 such that the deflection is about 30°.

Let the internal resistance of the cell be B, that of the galvanometer G, and that of the connecting wires C.

The latter may be determined by measuring the total length in use, and separately finding its resistance per metre. Then, if E denote the electromotive force of the cell—

$$\frac{E}{R_1+B+G+S} = k \tan \theta_1$$

$$\text{and } \frac{E}{R_2+B+G+S} = k \tan \theta_2$$

$$\text{hence } \frac{R_2+B+G+S}{R_1+B+G+S} = \frac{\tan \theta_1}{\tan \theta_2}$$

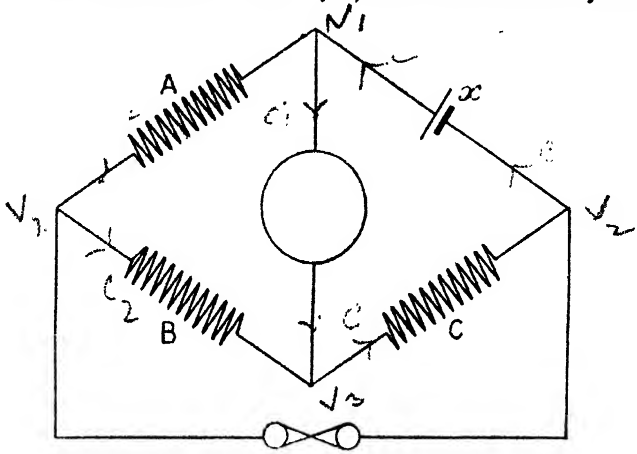
$$\text{and } B = \frac{(R_1+G+S)\tan \theta_1 - (R_2+G+S)\tan \theta_2}{\tan \theta_2 - \tan \theta_1}$$

If a reflecting instrument is used, scale readings may be substituted for the tangents.

118.

To determine the Internal Resistance of a Cell by Mance's Method.—The cell is placed in one arm of a bridge, a key taking its place in what is usually the battery branch. The galvanometer has its usual position.

The resistances in arms A, B, and C are then adjusted



until the deflection of the galvanometer is the same whether the key k be open or closed.

This condition being obtained—

$$x = \frac{AC}{B}$$

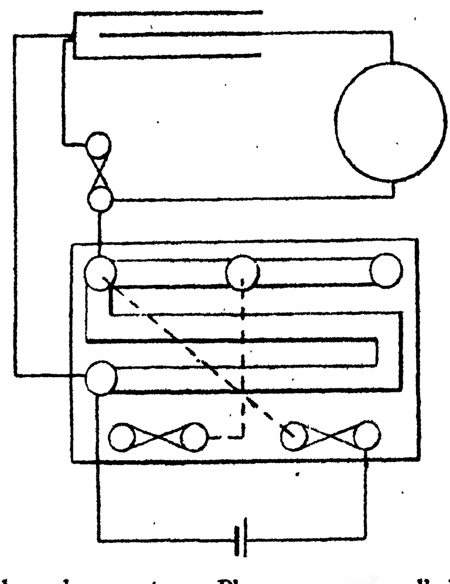
For it is only when this condition exists that no current passes through the branch containing the key when the latter is closed.

119.

To determine the Internal Resistance of a Cell by a Ballistic Method.—Let the cell be connected to an adjustable resistance, capable of carrying moderate currents, through a key.

Across the terminals of the resistance is connected a condenser having a ballistic galvanometer in circuit, and a key, k, or discharging.

Let the inserted resistance be infinite, and the first throw of the spot or needle be θ_1 when k is closed. The throw θ_1 is produced by a quantity of electricity sufficient to charge the condenser up to the open circuit E.M.F. of the cell passing



through the galvanometer. Plugs are now pulled out until upon depressing k about half the previous deflection is given. Let R be this resistance, and θ_2 the first throw of the needle. This is produced by a quantity of electricity which raises the condenser to a potential v. If V be the open circuit E.M.F. of the cell—

$$v = \frac{R}{R+B+C}V$$

$$\text{or } \frac{v}{V} = \frac{\theta_2}{\theta_1} = \frac{R}{R+B+C}$$

where B is the cell resistance, C that of the connections.

$$\text{Hence } B = \frac{\theta_1}{\theta_2}R - R - C = R\left(\frac{\theta_1 - \theta_2}{\theta_2}\right) - C$$

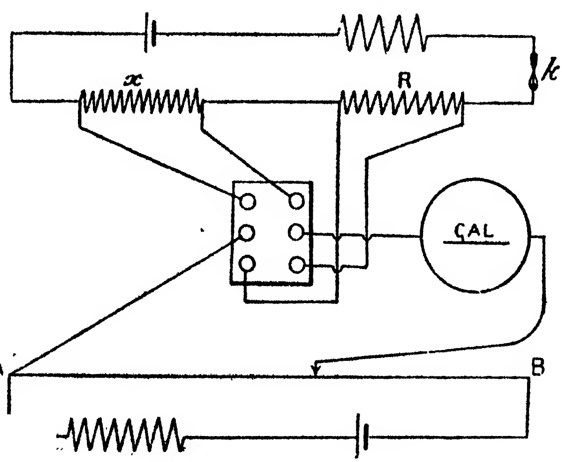
and C when relatively small may be neglected.

The condenser must be discharged between each observation of deflection.

120.

To determine a Resistance by Means of the Potentiometer.—The principle of this instrument and the method of comparing electromotive forces by its means have been dealt with in Art. 108.

If two resistances be connected in series with a cell, since



the same current flows in each, the differences of potential between their ends are proportional to their resistances.

The resistance x to be determined, a standard resistance R, an adjustable resistance for regulating the current, and a key, k, are connected as shown. The ends of x and R are connected to mercury cups. The rocker cups are connected to the potentiometer shunt circuit. By this means the potential

drops down x and R may successively be balanced on the potentiometer wire AB. Hence the resistances are proportional to the distances along AB at which a balance exists. Let x balance at m, R at n divisions. Then $x = \frac{m}{n}R$.

The key k should be closed before the rocker is put in the mercury cups.

By using a sensitive galvanometer, making the current through x and R large, and the fall of potential down AB small, very small resistances may be determined in this manner.

Since this is a null method, the resistance of connecting wires need not be taken into account; thermo-electric effects at points of connection should, as far as possible, be prevented by keeping them at uniform temperatures.

This method is also applicable to the measurement of electrolyte resistances, the details of which are dealt with in Art. 125.

121.

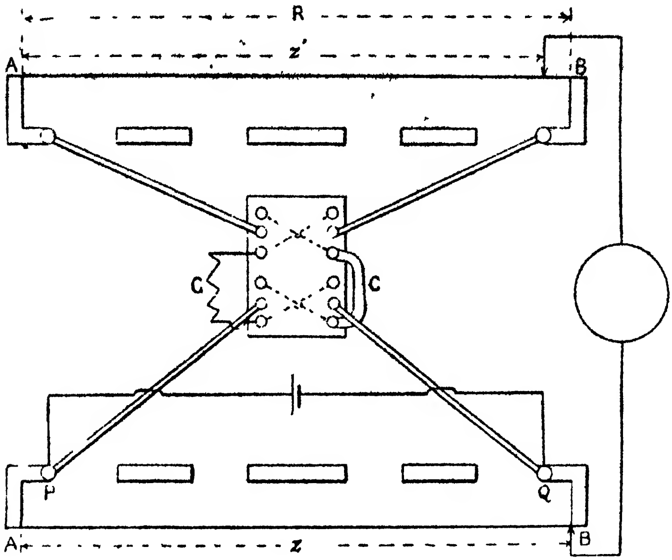
To calibrate the Wire of a Metre Bridge.—(*a*) *By Carey Foster's Method.*—Two bridges are required, and a switchboard of the form shown in which the circles represent mercury-cups, which are cross-connected. A rocker stands in the central pair of each group, and connects them either with the pairs above or below. The mercury-cups should be connected to terminal screws, and the base of the switch may be of ebonite or paraffin.

A resistance, G, is required, equal to that of a few centimetres' length of the bridge wire. This is termed the "gauge."

A thick conductor, C, of very low resistance is also used. This is termed the "connector."

The ends of the bridge wires are connected to the switchboard, and the contact-sliders connected together through a sensitive galvanometer. The gauge being switched on to the left hand of the bridge, the contact-piece on the lower wire is moved close up to its right-hand end. The upper one is then moved until a balance is obtained at some point p'.

Both rockers are now turned over into the upper holes, interchanging thus C and G. The upper slider being stationary, the lower is now moved to the left until a balance again exists.



The rockers are then put down, and, the lower being unmoved, the upper slider is balanced.

Proceeding thus, the entire length of each wire may be traversed.

Results should be entered thus:

Reading of first bridge.	Difference.	Reading of second bridge.	Difference.

The difference column gives that between the last two readings of the column to its left.

These differences are the lengths of the respective bridge wires which have the same resistance.

For let R be the resistance of the lower wire, z be that of the length between the left-hand end and the slider. Let R and z' denote the same for the upper wire. Let b and b' represent the resistance of the connections, bars, etc., between Q and B, Q and B′; also a and a' between P and A, and P′ and A′. Then, originally—

$$\frac{a+z}{G+a'+z'} = \frac{b+R-z}{b'+C+R'-z'}$$

After interchanging and balancing, z becoming some value z''.

$$\frac{a+z''}{C+a'+z'} = \frac{b+R-z''}{b'+G+R'-z'}$$

Hence—

$$z - z'' = (G-C)\frac{a+b+R}{a'+b'+G+C+R'}$$

which is constant.

At the ends of the wires the last two positions of balance may not be obtainable. To find the value of these portions, the gauge may be shortened until balance is given with the slider in each case at the end of the bridge wire. The resistances of these lengths have then the same ratios to that of the other lengths that the lengths of gauge wire have to the standard length.

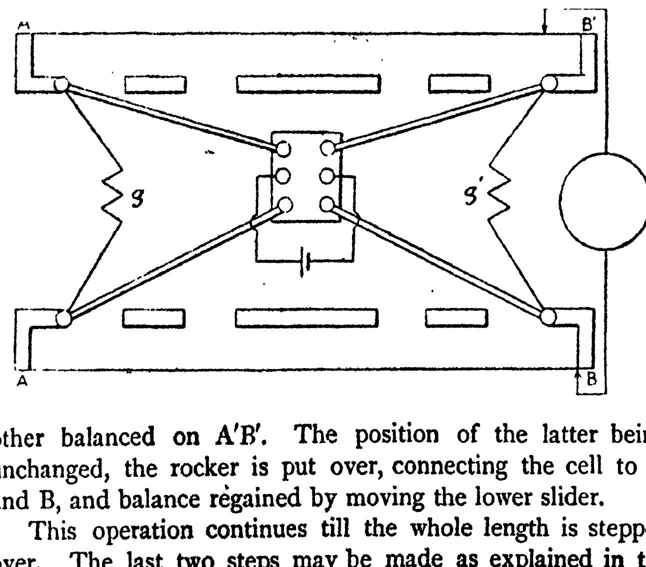
The total resistance of the bridge wire is then found, and hence that of each stepped-off portion. Dividing this by the length of each, the resistance of unit length in every part of the wire is found, and should be tabulated:

Divisions of wire between which unit length has value.	Value of resistance per unit length of wire.

(*b*) *By T. Gray's Method.*—The metre bridges are connected as shown, g and g' being two equal small resistances. This equality may be ensured by adjusting them until when the galvanometer is balanced for one position of the rocker, it remains so on interchanging them.

As the rocker requires to be of a special pattern, its place may conveniently be taken by two short wires bridging from cup to cup.

To calibrate, the rocker is put over so as to connect the cell to A and B′; the lower slider is placed close to B, and the



other balanced on A′B′. The position of the latter being unchanged, the rocker is put over, connecting the cell to A and B, and balance regained by moving the lower slider.

This operation continues till the whole length is stepped over. The last two steps may be made as explained in the previous method. Retaining the same signification for R, R′, z, and z', let a and a' denote the resistances of the left-hand bridge blocks, b and b' those on the right-hand end. Then, originally—

$$\frac{a+z}{g+b+R-z} = \frac{a'+g+z'}{b'+R'-z'},$$

and after switching over and balancing, z becoming z''—

$$\frac{a + z'' + g}{b + R - z''} = \frac{a' + z'}{b' + g + R' - z'}$$

Hence—

$$z - z'' = g\left(\frac{a + b + g + R}{a' + b' + g + R'} + 1\right)$$

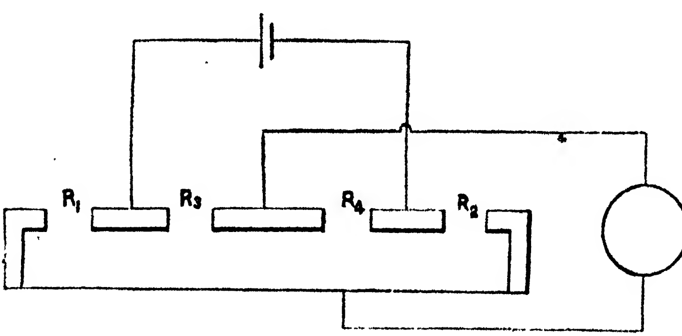
which is constant.

The total resistance of the wire should then be found, and the resistance for each centimetre length tabulated as explained above.

It will be noted that the connections shown in the diagrams as thick bars may equally well be of ordinary wire.

122.

To determine Small Differences in Resistances by Carey Foster's Method.—A metre bridge, the switchboard described in Art. 121, two resistances, R_3 and R_4, about equal to those which are to be compared, and R_1, R_2, those to be com-

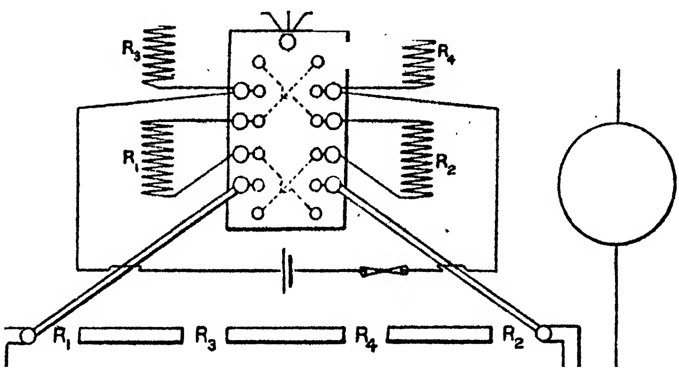


pared, are connected as shown. The positions which these would occupy if placed directly in the arms of the bridge are indicated, and the cell would be connected between R_1 and R_3, R_2 and R_4. The switch-board permits of R_1 and R_2 being interchanged in position. All the resistances should, if possible, be surrounded by water-jackets, as, if the temperature vary, the determinations possible by this method are valueless.

The connections between the bridge and switch should be stout bars.

In obtaining a balance, precautions must be taken against any thermo-electric currents. Error from this cause may be avoided by putting a commutator in the battery circuit, and balancing for both directions of the current.

A balance is obtained for one position of the switches, which should both be rocked over in the same direction, and the reading n_1 of the bridge noted. They are then put over, interchanging the positions of R_1 and R_2 relatively to the



bridge, and the new position of balance found. Let this be n_2 on the wire. Then the difference of resistance between R_1 and R_2 is equal to that of the bridge wire between divisions n_1 and n_2.

For let R be the resistance of the bridge wire, z_1 the resistance of that part of it between the left-hand end and the first position of balance n_1, z_2 between the same and the position n_2. Let a and b be the connection resistances at the left and right-hand ends.

$$\text{Then } \frac{R_3}{R_4} = \frac{R_1 + a + z_1}{R_2 + b + R - z_1}$$

and after interchanging—

$$\frac{R_3}{R_4} = \frac{R_2 + a + z_1}{R_1 + b + R - z_2}$$

$$\text{Hence } \frac{R_3}{R_3 + R_4} = \frac{R_1 + a + z_1}{R_1 + R_2 + a + b + R} = \frac{R_2 + a + z_2}{R_1 + R_2 + a + b + R}$$

$$\text{and } R_1 - R_2 = z_2 - z_1$$

The value of the resistance $z_2 - z_1$ between divisions n_2 and n_1 may be found from the calibration curve of the wire.

Two post-office boxes connected in series with the lower cups, and having the galvanometer connected between them, may be used instead of the metre bridge.

123.

To determine the Temperature Coefficient for the Resistance of a Material.—The arrangement and method of the last article should be employed. The material should be twisted into a coil, and inserted into a test-tube of oil. The connections to the switchboard should have a negligibly small resistance, or it should be found and deducted. The resistance of the coil of material should be approximately equal to that of some standard which may be used as the interchangeable resistance. The test-tube is firstly surrounded by melting ice, and the resistance, R, found. It is then placed in a vessel of water which is heated to 100°, and the resistance is again measured. Let it be R′.

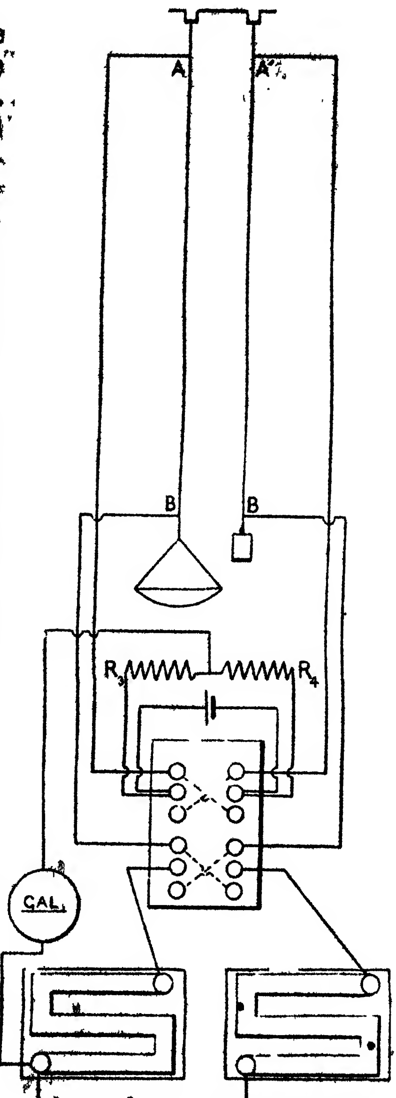
Then the temperature coefficient is $\dfrac{R' - R}{100R}$.

If ice be not available, the resistance may firstly be found at the observed temperature $t°$ of the air. The coefficient is then approximately—

$$\frac{R' - R}{(100 - t)R}$$

124.

To determine the Strain Coefficient for the Resistance of a Material.—The method of Art. 122 should be used for measuring the resistance. The wire to be investigated and a similar one of approximately equal resistance should be suspended as near together as possible, and protected from draughts. The resistances of the connections should be as small as possible. One wire should be attached to a scale-pan and the other loaded sufficiently to keep it taut. Weights should then be put in the pan, and the corresponding variations of resistance found by comparison with the other, which is constant. This should be done for both increasing and diminishing loads. By measuring the diameter of the wire with the micrometer gauge, its sectional area, and the stress per square centimetre of the same, should be found.



Results should be tabulated thus:

Load.	Stress.	$n_2 - n_1$.	$\frac{R' - R}{R}$.

where $n_2 - n_1$ is the length of bridge wire between the positions of balance when a metre bridge is used, $z_2 - z_1$ the difference of resistance between the two post-office boxes, or that of the wire between n_1 and n_2, R the original unloaded resistance, R′ the resistance at the load of the entry. To determine R, the resistance of the auxiliary wire should be found as a preliminary by one of the usual methods.

The diagram shows the actual connections when post-office boxes are employed, and those necessary for a metre bridge are apparent from the diagram of Art. **122**.

The most satisfactory metal to investigate is iron, and the wires should be between one and two metres long.

A curve should finally be plotted having stress as abscissæ and increase of resistance as ordinates.

125.

To determine the Specific Resistance of an Electrolyte.—The ordinary methods of determining a resistance fail in the case of electrolytes, owing to the polarization at the electrodes. The following means are, however, applicable:—

(*a*) *Horsford's Method.*—A glass tube of known internal diameter is fitted at its ends with corks through which pass stout wires carrying discs at their inner ends. These discs may thus be adjusted at various distances apart. To allow any liberated gas to escape, the tube should be clamped vertically, and the upper cork bored with an additional hole. The resistance of a column of length L is found in the usual way by a bridge. The discs are then brought nearer together,

so that the length of columns is L′, and the resistance again found. The difference of these resistances is taken as that of a column L − L′ in length. Dividing this by L − L′ and multiplying by the sectional area of the tube, the specific resistance is found.

The discs should be of thick platinum, and previously blackened by having a current sent in reverse directions several times between them while immersed in a solution of platinum tetrachloride, after which they should be washed in water. Platinum wires should be welded to them before this operation; that of the lower disc may pass directly through the cork, that of the upper adjustable one should be sealed into a fine glass tube sliding in the cork. Contact with this last is made by pouring clean mercury in, and passing down an ordinary wire.

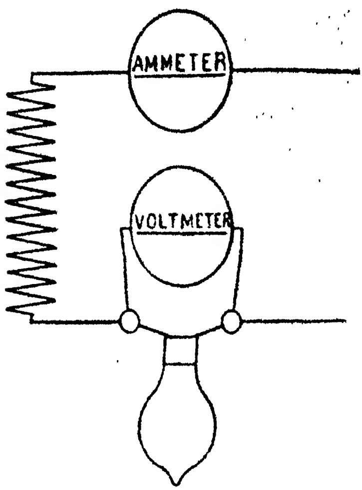
(*b*) *Branly's Method.*—This consists in measuring the difference of potential at two points on a liquid column, and noting the current flowing through it. The resistance is then equal to the potential difference divided by the current.

The potential electrodes should be platinum wires sealed in tubes. In order to prevent polarization their potential difference should be found by a potentiometer (Art. 108), that occurring at the main current electrodes introduces no error.

(*c*) *Kohlrausch's Method.*—The ordinary metre bridge is here used, but an alternating current is employed. The secondary of a small induction-coil is connected to the ends of the bridge wire, and a telephone receiver used in place of the galvanometer. The two positions on the wire at which the telephone just becomes inaudible are noted, and the mean position taken as that at which balance occurs. It is desirable to have the coil itself as inaudible as possible, and to make the determination in a quiet room. The usual ratios exist at the position of silence, and, the length and sectional area of the electrolyte column being found, its specific resistance may be calculated.

126.

To determine the Resistance of a Lamp for Various Values of its Current.—As the resistance of carbon is reduced by rise of temperature, that of a lamp diminishes with increase of current.



The resistance should be found when the lamp is cold by means of a Wheatstone's bridge.

It should then be connected in series with a carbon or lamp resistance and an ammeter, and connected through a key to the main lighting leads.

Across the terminals of the lamp a voltmeter is connected. The resistance should be reduced by successive steps, the instruments being read for each. The lamp resistance for each equals the voltmeter indication divided by that of the ammeter.

Results should be tabulated thus:

R of lamp cold =

Terminal volts.	Current.	Resistance.

A curve should be plotted with currents as abscissæ, and resistances as ordinates.

Capacities.

The capacity of a body is the numerical value of the charge which raises it to unit potential, *i.e.* in electro-magnetic units, the number of coulombs raising it to a potential of one volt.

A condenser brought to this potential by one coulomb has a capacity of one farad. Such a one would be of inordinate size, and a few microfarads is the usual limit.

If in a condenser the dielectric is not a gas, a certain portion of the charge is gradually absorbed, lowering the potential. Hence a certain amount of indefiniteness is introduced in measurements of capacity. This same portion of the charge, after the condenser is discharged, returns to the plates, giving them a residual charge. The amount of this may be measured by again discharging through a galvanometer, or observing the potential, some time after the first discharge.

127.

To compare Capacities by the Electrometer, and to find the Relative Capacity of the latter's Quadrants.—The latter determination is advantageously made as a preliminary to comparisons between other capacities. For this purpose a condenser of very small capacity, 0·001 microfarad or so, is required. A parallel-plate condenser of suitable size is readily made, for which the capacity may be calculated electrostatically, and so found in farads by dividing by 9×10^{11}.

The electrometer needle is then charged to a high potential, and the quadrants connected by wires, which should be as short as possible, to the condenser plates. These should be charged by momentary contact with the terminals of a battery of several cells, and the deflection noted. Let it be θ_1.

The condenser and quadrants are now short-circuited and discharged by touching the electrometer terminals with the two ends of a wire. The wires are then disconnected from the condenser terminals, and the latter alone is charged by contact

with the battery wires, after which the electrometer wires are then, by means of pieces of ebonite rod, moved into contact with the condenser plates, and the consequent deflection noted. Let this be θ_2.

Then if Q be the charge received by the condenser alone, K the latter's capacity, and k that of the quadrants—

$$\frac{Q}{K} \propto \theta_1$$

$$\text{and } \frac{Q}{K+k} \propto \theta_2$$

$$\text{Hence } k = K\frac{\theta_1 - \theta_2}{\theta_2}$$

The value of k being thus known, two condenser capacities may be compared by connecting one of them to the electrometer and charging it from the battery. Let the deflection be θ, and the capacity of this condenser be K_1.

The other condenser is then connected to the first momentarily, by touching the terminals of the first with wires attached to those of the other. Let the deflection fall to θ'.

$$\text{Then } \frac{Q}{K_1+k} \propto \theta$$

$$\text{and } \frac{Q}{K_1+K_2+k} \propto \theta'$$

$$\text{and } K_2 = \frac{(K_1+k)(\theta-\theta')}{\theta'}$$

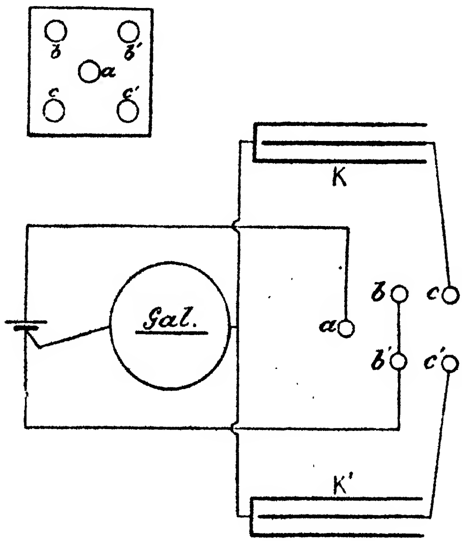
In approximate work k may be neglected.

The above equations have taken no account of the capacity of connecting-wires. This capacity, always small, should be made a minimum by using short connections. The wires should bridge completely from terminal to terminal, or be supported by their own rigidity when screwed down at one end so as never to come in contact with surrounding objects.

128.

To compare the Capacities of Condensers by the Ballistic Galvanometer.—The principle of the method is that when two condensers are charged to the same potential, their charges are proportional to their capacities, and consequently, if, when charged or discharged, these quantities of electricity are sent through a ballistic galvanometer, the deflections indicate the capacities.

Neglecting correction for damping, the capacities are



actually proportional to the sine of half the angle of the first swing; but if the galvanometer be a reflecting instrument they are approximately as the scale readings.

For opening and closing the circuits, a Kempe's key is very convenient.

This contains two keys so arranged that, upon depressing either, contact is made between *a* and *c* or *c′*, when released contact is made between *b* and *c* or *b′* and *c′*. An eccentric is also fitted by which either key may be fixed in an off position.

In the absence of such a key an ebonite or paraffin plate with mercury cups, as shown, may be used, contact being made by a bent wire dipping into any pair of cups desired.

The key being originally across *b* and *c*, it is depressed on to *a* and *c*, charging K through the galvanometer. The throw is noted. When the spot has come to rest the key is allowed to rise, connecting *b* and *c*, and discharging K. The throw is now also noted. Let the mean throw be θ. Repeating the operations with the other key, let the mean throw be θ'. Then—

$$\frac{\text{capacity of K}}{\text{capacity of K}'} = \frac{\theta}{\theta'}$$

The contacts of the keys should be carefully cleaned by gently pressing and rubbing a card between them. The ebonite legs also should be dry and clean.

It is obvious that if the same condenser be charged from various sources of electromotive force, the galvanometer throws will be proportional to the E.M.F.'s, and hence that they may be compared by this means.

129.

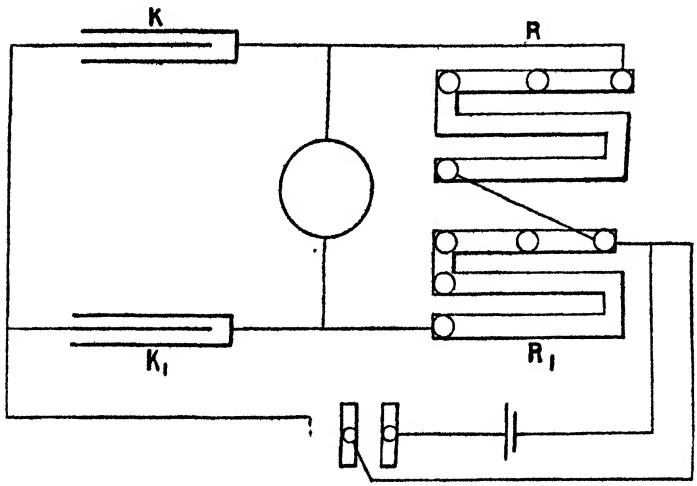
To compare the Capacities of Condensers by a Bridge Method.—In this, which has the advantage of being a null method, two post-office boxes are placed in two arms of a bridge, the condensers to be compared in the other two.

A Morse or Kempe's key is placed in the battery circuit and connected as shown. The galvanometer should be sensitive and of high resistance, while the battery should give twenty or more volts.

The resistance R should be made two or three thousand ohms, and R_1 adjusted until, on depressing the key and charging the condensers, no ballistic throw is produced.

In this case the potential of each end of the galvanometer wire, which practically coincides with that of the adjacent plates of the condensers, is the same throughout the period of

charging. Consequently each condenser in any small interval of time must receive a charge proportional to its capacity.



But the quantities of electricity passing through R and R_1 in any small interval of time are proportional to $\frac{1}{R}$ and $\frac{1}{R_1}$; hence–

$$\frac{K}{K_1} = \frac{R_1}{R}$$

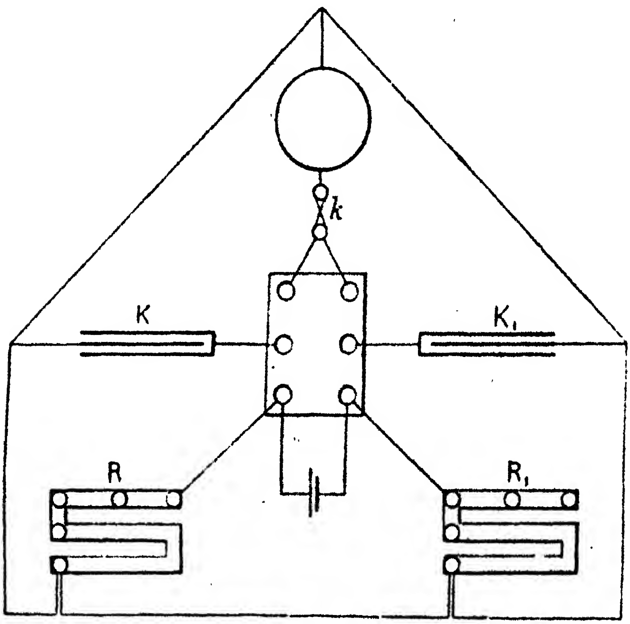
130.

To compare the Capacities of Condensers by a Method of Mixtures.—The two condensers and resistance boxes are connected, as shown, to a switch, with which the galvanometer is also connected through a key, *k*.

The resistances R and R_1 are then adjusted until no deflection occurs upon putting the rocker into the lower cups and charging the condensers; rocking it over to the upper ones and connecting them together; then depressing the galvanometer key.

Under these conditions the charges in the two condensers are equal and opposite, so that when connected they neutralize each other, and nothing remains to flow through the galvanometer.

But the potential difference between the plates of K is CR_1 when charged; and of K_1 it is similarly CR_{11}, if C be the



current flowing through the resistances. Hence the respective charges are KCR and K_1CR_{11}, and these are equal, therefore—

$$\frac{K}{K_1} = \frac{R_1}{R}$$

An electrometer may be used in place of the galvanometer, the connection being made to alternate quadrants. In this case the resistances are similarly adjusted until there is no deflection upon depressing the key *k*.

131.

To determine the Capacity of a Condenser. — If a quantity of electricity Q be sent through a ballistic galvanometer, reading from a mirror and scale—

$$Q \text{ in coulombs} = \frac{10HT}{\pi G} \sin\frac{\theta}{2}\left(1 + \frac{\lambda}{2}\right) = \frac{10HT}{\pi G} \cdot \frac{l}{2L} \text{ approximately}$$

where H = horizontal component of the earth's magnetic force;
T = period of the needle's complete swing;
G = the galvanometer constant;
θ = angle through which needle first swings when discharge passes through galvanometer;
λ = the logarithmic decrement;
l = distance which the spot first moves over the scale;
L = distance between needle and scale.

If the deflection has been produced by the charge of a condenser—

$$Q = VK$$

where V is the potential, and K the capacity of the condenser.

$$\text{Hence } K = \frac{10HTl}{2\pi GVL}$$

The period T of the needle should be found, and l noted when the condenser charge is passed through the galvanometer (Art. 101).

The cell used to charge the condenser is then connected in series with a resistance box and the galvanometer, which should now be shunted. The resistance is adjusted until some convenient permanent displacement l' of the spot is produced. Let R be the resistance inserted, G′ that of the galvanometer, S that of the shunt. Then a total resistance R′, where—

$$R' = \left(R + \frac{G'S}{G'+S}\right)\frac{G'+S}{S} = \frac{R(G'+S)}{S} + G'$$

would have given the same deflection, l', with the unshunted galvanometer.

$$\text{Now, } \frac{V}{R'} = \frac{Hl'}{2GL} \times 10$$

$$\text{and } \frac{1}{R'} = \frac{Hl'}{2GVL} \times 10$$

Substituting in the expression for K given above—

$$K = \frac{T}{\pi R'} \cdot \frac{l}{l'} \text{ farads}$$

132.

To determine the Specific Inductive Capacity of a Solid Dielectric.—The specific inductive capacity of a dielectric is the ratio of the capacity of a condenser having that dielectric between its plates to the capacity of the same condenser having air between them.

In cases where the dielectric is obtainable in the form of a flat slab of uniform thickness, the following method of determining its specific inductive capacity may be employed.

A condenser consisting of two metallic discs, the one of which is supplied with a guard-ring, is connected to an electrometer, one plate to each terminal. The upper plate and guard-ring are suspended coaxially over the lower plate by a silk thread, which permits the distance between them to be varied. Each plate is separately adjusted in a horizontal plane by a level to ensure their being parallel to each other, and their distance apart is made such that the slab of dielectric may be subsequently inserted between them without touching either.

A mark is made upon the suspending thread, and a cathetometer focussed upon it, after which the condenser plates are momentarily put in contact with the poles of a battery or cell, and both electrometer and cathetometer readings are taken.

The dielectric slab is now inserted between the condenser plates, thus reducing the electrometer reading, and the latter is brought up to its original value by increasing the distance

between the plates. The new position of the mark is read from the cathetometer, and the distance, d, that the plates have been withdrawn is found. Then, if t be the thickness of the slab of specific inductive capacity K—

$$d = t\left(1 - \frac{1}{K}\right)$$

$$K = \frac{t}{t - d}$$

133.

To determine the Specific Inductive Capacity of a Liquid Dielectric.—(*a*) The method of Art. 132 may be applied to a liquid if the latter be contained in a flat tray capable of insertion between the condenser plates. The condenser is charged with the empty tray between the plates to a noted potential, and the position of the movable plate observed. The dielectric is then poured into the tray to a noted depth, t. The fall of potential produced is counteracted by drawing the plates apart, and when it arrives at the original value the position of the upper plate is again noted.

Let d be the difference between the two positions of the plate, K the specific inductive capacity. Then—

$$d = t\left(1 - \frac{1}{K}\right) \text{ and } K = \frac{t}{t - d}$$

(*b*) *By Silow's Inductometer.*—This consists of a glass jar with four tin-foil strips, separated from each other by spaces about a centimetre wide, pasted internally. A needle with two semi-cylindrical faces is carried inside these from a torsion head. The opposite pairs of foil strips are connected together and to terminals, to one of which the needle is also connected. The terminals are connected to the poles of a battery, or other points, between which a constant potential difference exists, and the deflected needle brought back to zero by turning the torsion head.

Let θ be the angle through which it is turned. The liquid dielectric is now poured in, and the operations repeated, giving some angle of twist, θ'.

The specific inductive capacity is then equal to $\frac{\theta'}{\theta}$

134.

To determine the Current producing one Scale Division Permanent Deflection, and the Quantity of Electricity producing the same Ballistic Throw, for a Given Galvanometer, by Means of a Cell of known E.M.F. and Standard Resistances.—The resistances B and G′ of the cell and galvanometer are found by the method of Arts. **119, 113.** The galvanometer is shunted by a resistance S, and the number of divisions over which the spot is displaced with a resistance R in circuit noted. The current is reversed, and the mean reading l taken.

If the galvanometer had not been shunted, the same deflection would have been given with a total resistance R′ in circuit, where—

$$R' = \frac{G'+S}{S}\left(R + B + \frac{G'S}{G'+S}\right)$$

If V be the electromotive force of the cell, one scale division displacement is produced by a current—

$$\frac{C}{l} = \frac{V}{R'l}$$

The quantity of electricity producing one division ballistic throw is—

$$Q = \frac{10HT}{2\pi GL}$$

where the letters have the same meaning as in Art. **131**;

$$\left.\begin{array}{r}\text{and the current producing}\\ \text{unit steady deflection}\end{array}\right\} = \frac{10H}{2GL}$$

$$\text{Hence } \frac{10H}{2GL} = \frac{V}{R'l}$$

$$\text{and } Q = \frac{VT}{\pi R'l}$$

The only determination, therefore, beyond finding $\frac{V}{R'l}$, is to observe T, the period of swing.

Inductance.

The inductance of a circuit is the number of magnetic lines due to its own current which pass through it per unit current. The same line passing through several loops of the circuit is to be counted as many times. The self-inductance of a circuit is usually denoted by L.

The mutual inductance of two circuits is the number of magnetic lines which, in the sense explained above, pass through one of them per unit current in the other.

This number is the same for both circuits, irrespective of which carries the current, and is usually denoted by M.

In the case of circuits remote from magnetic media, L and, for the same relative position of the two circuits, M are constants. If, however, the circuits be near or encircle magnetic masses, the inductance varies with different values of the current, owing to variation in the permeability of the masses. It also varies to a less degree for the same current, owing to hysteresis.

The unit of inductance is the secohm, equal to 10^9 absolute C.G.S. units. It is of importance in calculations respecting varying currents, as an electromotive force is produced in a circuit of amount $L\frac{dC}{dt}$, where C denotes the current.

Another unit, the Henry, is also employed. It equals

10^9 C.G.S. units. This represents the number of lines produced by a circuit, each line being counted only once, irrespective of it passing through loops more frequently. Thus in a solenoid it is the number of lines passing through, and equals the inductance for the same solenoid expressed in secohms divided by the total number of turns.

The moment of a coil is the twisting moment about an axis passing through the centre of its figure at right angles to its axis, when the coil carries unit current and lies in a uniform field of unit strength, the lines of which are at right angles to its own.

This moment is equal to the inductance of the coil, expressed in Henrys, divided by 4π; or to the same, expressed in secohms, divided by 4π times the number of turns in the coil.

This expression for the moment applies to flat coils, but in the case of solenoids must be multiplied by their length, the assumption in each case being that the radial depth of winding is negligible compared with the mean radius.

In the determinations of inductance precaution must be taken that the galvanometer, when in position and disconnected, is uninfluenced by the currents in any of the circuits.

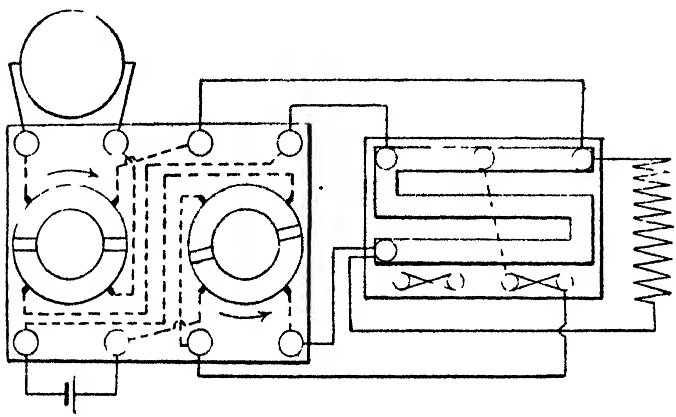
The sensitiveness of the following methods may be increased by (*a*) reversing the cell connections by a suitable commutator instead of merely opening or closing the circuit; (*b*) employing a secohmmeter. This instrument consists of two cylindrical commutators keyed to the same spindle. The circumference of each is divided into two parts, insulated from each other by narrow strips. Against these commutators press brushes, making connections with the cell, galvanometer, and bridge in such a way that when they are rotated, connection between the last two is made at any desired interval after opening connection between the first and last. This interval is adjusted by altering the angular position of one commutator on the spindle relatively to the other.

The connections are so arranged that both make and break effects act in the same direction on the galvanometer,

* By many writers, Henry and secohm are used synonymously for the unit here termed the secohm.

giving a permanent deflection instead of a ballistic throw when the secohmmeter spindle is rotated.

The method of coupling up an ordinary bridge is as shown, dotted lines indicating internal connections.



135.

On the Production of Induced Currents.—The relation between the direction of deflection and that of the current passing through it should firstly be found for a moderately sensitive galvanometer by connecting a cell through a large resistance to its terminals, and noting the direction of the deflection and of the current. The resistance should be as large as possible at first, and gradually reduced till the deflection is observable.

The following may then be investigated :—

(*a*) The induced currents produced in coils by the motions of a magnet.

The cell and resistance being removed from the galvanometer, the ends of a coil are connected to the latter. The north-seeking pole of a magnet is then quickly inserted into the coil, the effect on the galvanometer noted, and the direction of the current found. The latter may be expressed as "clockwise" and "anti-clockwise," according to the direction

in which it flows round the coil when viewed from the end or face at which the magnet is inserted.

When the galvanometer needle has come to rest, the magnet should be quickly withdrawn from the coil, and the same observations made.

The same operations should be repeated with the south-seeking pole of the magnet.

Results should be tabulated thus:

N. pole inserted.	S. pole inserted.
Direction of current—	Direction of current—
N. pole withdrawn.	S. pole withdrawn.
Direction of current—	Direction of current—

The effect of bringing either pole from a considerable distance level with, but not actually inserted in, the coil should be noted, also that of its withdrawal from this position, and that of passing either pole quickly across the end or face of the coil.

The connection between the directions of the magnetic lines due to the current and those of the magnet should be considered.

(*b*) Similar operations should be performed with a coil connected to a cell in place of the magnet, the relative directions of the currents in the coils being noted when both are viewed in the same direction.

(*c*) If solenoids, one coil should now be placed inside the other, assuming they are of unequal diameters; or, if flat coils, they may be placed side by side co-axially. The direction of the current produced in that connected to the galvanometer should be noted when the other is connected to, or disconnected from a cell, "clockwise" or otherwise being as viewed from the side on which the inducing coil lies. With solenoids, the directions may be termed "direct" and "reverse." The connection and disconnection should be made by a key in the cell-circuit.

136.

To determine the Connection between the Magnitudes of the Current Variation in the Primary and the Electromotive Force in the Secondary of a Pair of Coils when their Relative Position is unchanged.—When, as in Art. **135** (*c*), the induced electromotive force in one coil is produced by the alteration in the current flowing in another, the former coil is termed the "secondary" and the latter the "primary."

Let the coils be arranged as for (*c*), Art. **135**, and an adjustable resistance put in series with the primary and its cell. In this circuit should also be placed a key. The resistance should firstly be large, and gradually reduced until a deflection is given by the galvanometer in the secondary circuit, when the current in the primary is stopped by opening the key. Now the total quantity of electricity passing through the secondary coil is proportional to the magnitude of the impulsive electromotive force produced in it, and for brief currents of this nature the sine of half the angle of the galvanometer needle's first swing is proportional to the quantity of electricity sent through the instrument. Hence, if θ be this angle—

$$\sin \tfrac{1}{2}\theta \propto \text{E.M.F. in secondary}$$

the connection between primary current and secondary E.M.F. should be investigated by reducing the resistance in the primary by steps, and for each value opening the key, and noting both the resistance and the deflection produced on opening.

The resistance of the primary coil itself may afterwards be found.

Let this be R_1. If R_2 be the resistance put in by the box, and if the resistance of the cell, which is assumed to be of constant E.M.F., be neglected, as, being small, it may be, then the currents in the primary are proportional to $\frac{1}{R_1 + R}$.

Results should be tabulated thus:

R_2.	$R_1 + R_2$.	Current $\propto \left(\frac{1}{R_1 + R_2}\right)$	$\sin \frac{1}{2}\theta$.	$(\sin \frac{1}{2}\theta)(R_1 + R_2)$.

If the galvanometer be a reflecting instrument, the scale-readings may be taken instead of sin $\frac{1}{2}\theta$, without introducing any great error.

A curve should be plotted with values of the primary current $\left(\propto \frac{1}{R_1 + R_2}\right)$ as abscissæ, and of the secondary E.M.F. ($\propto$ sin $\frac{1}{2}\theta$) as ordinates.

137.

To determine the Relation between the Mutual Inductance of Two Flat Coils and the Distance they are Apart, the Coils being Co-axial and in Parallel Planes.—One coil being connected to the galvanometer, the other is connected through a resistance and key to a cell of constant E.M.F. They are then placed as stated, and brought nearer together until a deflection is given, when the primary circuit is opened. The deflection is noted, *i.e.* the angle of the first swing, and the distance the coils are apart measured.

They are then brought nearer together by steps, the deflection on opening circuit being noted for each position, and the distance measured.

The deflection on closing the primary circuit may be noted instead of that upon opening it, with some consequent saving of time.

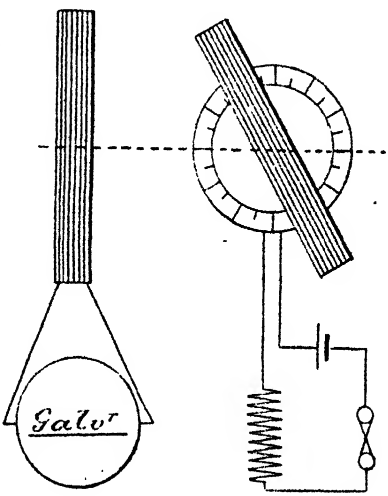
Results should be stated thus:

Distance between coils.	$\sin \frac{1}{2}\theta$.

A curve should be set out having distances apart of the coils as abscissæ and values of the mutual inductance ($\propto$ sin $\frac{1}{2}\theta$) as ordinates.

138.

To determine the Relation between the Mutual Inductance of Two Flat Coils, and the Angle between their Planes. —The two coils are connected as in Art. **137**, and placed co-axially in parallel planes, the distance between their faces being just greater than their radius. The resistance in the primary is now reduced until the galvanometer gives a considerable deflection upon opening or closing the primary circuit. This deflection, θ, is observed.



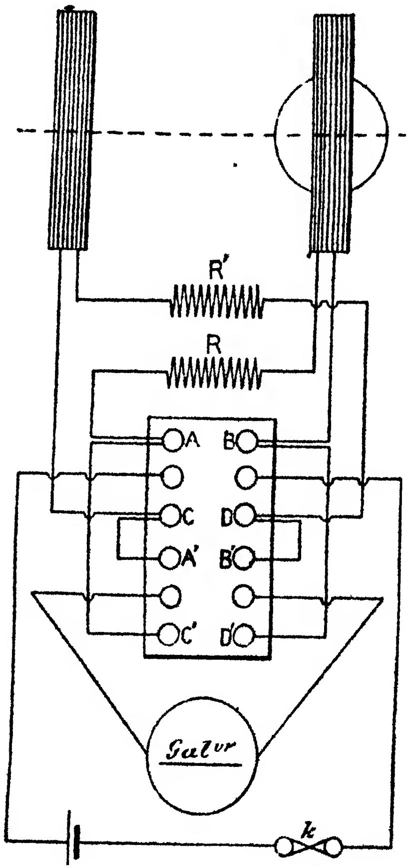
One of the coils should now be turned about either a vertical or horizontal axis through an angle. If the coil have a circle divided into degrees and an index, this should be 15°. If these be absent, an angle of 45° may be estimated. The deflection upon opening or closing the primary is noted for this position.

The angle is altered in this manner by steps, and the deflection at each noted, until the coil has been turned through 360°.

Results should be tabulated thus:

Angle between coils.	Mutual induction ($\propto$ sin $\frac{1}{2}\theta$).

By connecting each coil with an adjustable resistance, and with mercury switches, so that the cell and galvanometer may be switched from one coil to the other, it may be proved that



the mutual inductance is the same whichever coil be the primary.

Placing R and R' in series with the coils, and adjusting till R + resistance of its coil equals R' + resistance of its coil, the coil resistances being known, the switches are then put over to

AB, A′B′ and CD, C′D′ successively, and the deflection for each noted when the key k is opened or closed. This is repeated for each position of the coils.

In this case the results should be stated thus:

Angle between coils.	Mutual inductance ($\propto \sin \tfrac{1}{2}\theta$).	
	Coil at AB primary.	Coil at A′B′ primary.

139.

To determine the Relation between the Electromotive Force induced in a Circuit and the Rate at which it is cutting across Magnetic Lines.—The most convenient method of finding this relation is to excite the field coils of a small dynamo separately from a battery of constant E. M. F., and to rotate its armature by another motor, the speed of which may be adjusted. The brushes are connected to a suitable voltmeter, and the armature circuit is otherwise open.

The voltmeter readings should then be observed, and the speed simultaneously taken by some form of speed-counter.

Several forms of speed-counter exist. In one the revolutions per minute are instantaneously given by an index. As the index hunts considerably it is a somewhat unsatisfactory form. In others the total number of revolutions in an observed time is given by the difference of the original and final readings. In the best forms of this type a watch embodied in the counter is set to its zero by touching a spring, and stops automatically at the end of the observation. All are actuated in the same manner, viz., by pressing a pointed bar into a small depression in the end of the revolving spindle.

The circuit of the exciting coils should have an ammeter in circuit to indicate any variation in the number of magnetic

lines cut per revolution, a contingency which must be guarded against.

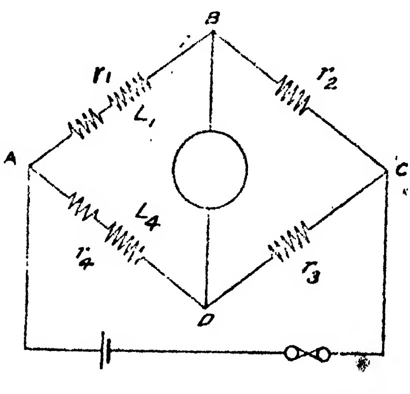
If, as is usually the case, the motor driving the dynamo be driven from constant voltage mains, an adjustable resistance must be inserted in its circuit, by varying which its speed may be altered.

A curve should be plotted with speeds as abscissæ and the induced electromotive force as ordinates.

140.

To compare the Inductances of Two Coils.—The two coils, each with a resistance-box in series, are placed in the two arms AB, AD of a bridge.

The galvanometer is connected to two alternate junctions of the arms, and the battery through a key to the others.



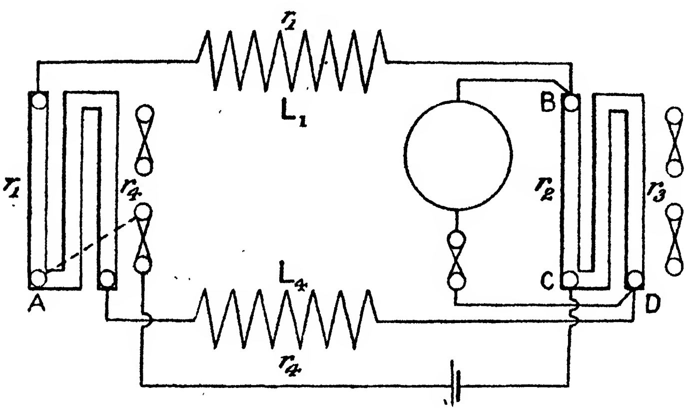
A balance is now found for steady currents, in which case $\frac{r_1}{r_4} = \frac{r_2}{r_3}$. If the battery key, however, be opened or closed, a brief current passes through the galvanometer owing to the inequality of inductive effects in the coils. But these effects will be equal when—

$$L_1 \times \text{steady current in ABC} = L_4 \times \text{steady current in ADC}$$

If the bridge be so adjusted that this is the case, while $\frac{r_1}{r_4}$ also equals $\frac{r_2}{r_3}$, it is balanced for both steady and varying currents. Consequently—

$$\frac{L_1}{L_4} = \frac{r_1}{r_4} = \frac{r_2}{r_3}$$

The operation is, therefore, to vary the total resistance of



ABC or ADC by means of the resistances in AB and AD, and to again adjust r_2 and r_3 for steady balance. The magnitude of the galvanometer deflection upon opening or closing the battery key relatively to that previously produced on doing the same, indicates whether the resistance has been varied in the right direction. The alteration has been correctly made if it reduces the throw; while if the latter is in the opposite direction to the first when operating similarly with the key, a correct but excessive alteration is indicated.

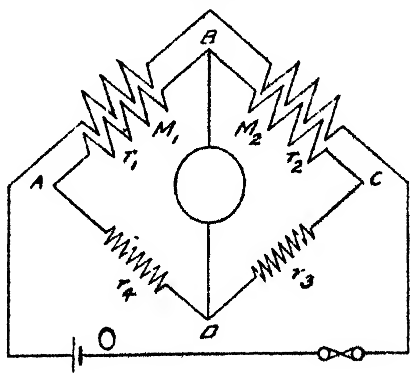
Proceeding in this way, altering the resistances in AB and CD, and balancing with r_2 and r_3 for steady currents, then opening or closing the key k, and noting the ballistic throw, the latter may be reduced to zero.

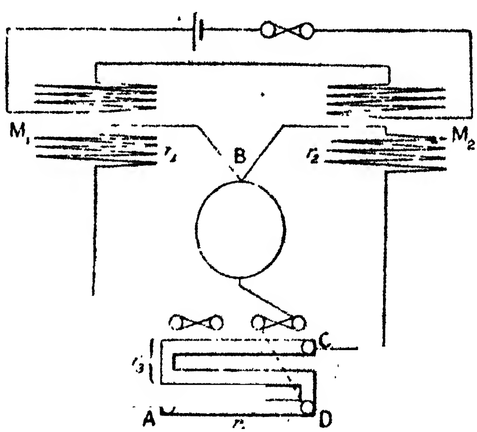
Then, as stated—

$$\frac{L_1}{L_4} = \frac{r_2}{r_3}$$

141.

To compare the Mutual Inductances of Two Pairs of Coils.—Two of the coils, one from each pair, are placed in series, and connected through a key to the battery.





The remaining two form two arms of a bridge, the other arms of which are non-inductive.

The galvanometer is connected from between the two coil-arms to the junction of the other two arms.

In general, upon opening or closing the battery key, a brief flow passes through the galvanometer, giving a ballistic throw.

The resistances r_3 and r_4 should now be adjusted until no deflection occurs upon opening or closing the key, and their values then observed.

The values of r_1 and r_2, *i.e.* the resistances of the coils should be determined for steady currents in the usual manner.

$$\text{Then } \frac{M_1}{M_2} = \frac{r_1 + r_4}{r_2 + r_3}$$

For let x be the current flowing through AB, y that flowing from B to D.

The electromotive forces between A and B, B and C are proportional to M_1 and M_2, since the same battery current flows through both pairs of coils.

Then in ABD—

$$r_1x + Gy + r_4x = M_1$$

and in BCD—

$$r_2(x - y) + r_3(x - y) - Gy = M_2$$

consequently where y equals o—

$$\frac{M_1}{M_2} = \frac{r_1 + r_4}{r_2 + r_3}$$

142.

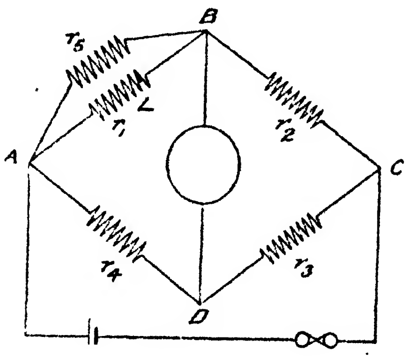
To determine the Inductance of a Coil by Comparison with a Resistance.—The coil of inductance L is placed in one arm of a bridge, the remaining arms of which are non-inductive. The galvanometer of resistance G is placed across two of the junctions of the arms, the battery across the other two, each having a key in circuit.

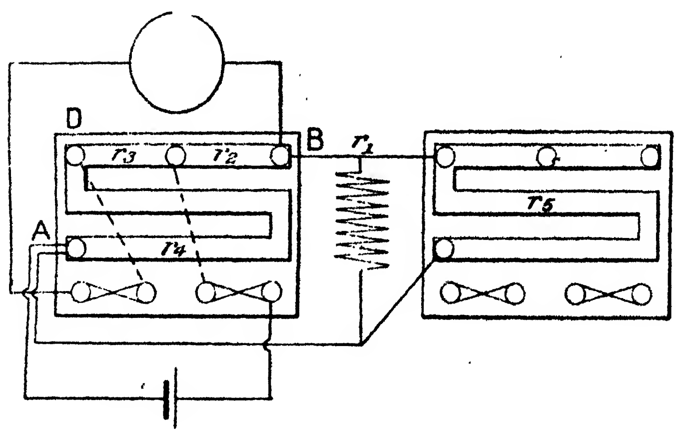
The coil being in AB, a resistance-box should be placed in parallel with it, and a plug for some hundreds of ohms drawn out. The bridge is then balanced for steady currents.

If now an electromotive force were inserted in AB, the

galvanometer would give a deflection proportional to the magnitude of that force; but the inductance during the rise or fall of current in AB produces an electromotive force, equal at any instant to $L\frac{d^2x}{dt^2}$, where $\frac{dx}{dt}$ is the current at that time.

The total quantity, y, of electricity passing through the





galvanometer is then due to an impulsive electromotive force equal to the integral of $L\frac{d^2x}{dt^2}$, the limits being $\frac{dx}{dt} = o$, and $\frac{dx}{dt} = c$, the steady current in AB. And $\int_0^c L\frac{d^2x}{dt} = Lc$.

Therefore by the theory of the ballistic galvanometer—

$$Lc \propto \frac{HT}{\pi G} \sin \tfrac{1}{2}\theta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

where T is the period of the galvanometer, and θ_1 the ballistic throw given when the battery key is opened or closed while the galvanometer key is closed.

The observation of this throw, θ_1, is now made.

A small change of resistance is now made in the arm AB by means of the resistance-box. Let this be R. This dis turbs the previous balance, giving some steady deflection, θ_2, which is noted.

Then the current in the galvanometer is due to a virtual electromotive force equal to—

$$c\left(\frac{(r_5 \pm R)r_1}{r_5 \pm R + r} - \frac{r_5 r_1}{r_5 + r_1}\right) = \frac{cr_1 R}{r_5 + r_1} \text{ approximately}$$

neglecting the small variation of c.

$$\text{Therefore } \frac{cr_1 R}{r_5 + r_1} \propto \frac{H}{G} \tan \theta_2 \quad . \quad . \quad . \quad (b)$$

and dividing (a) by (b)—

$$L = \frac{r_1}{r_5 + r_1} \cdot \frac{RT}{\pi} \cdot \frac{\sin \frac{1}{2}\theta_1}{\tan \theta_2}$$

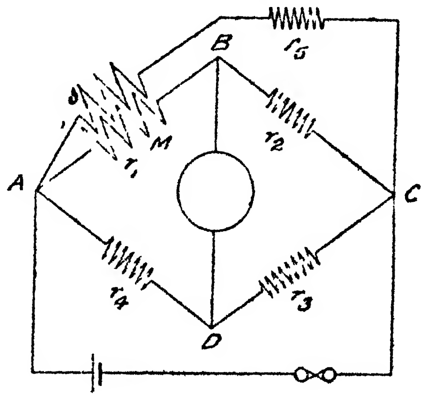
143.

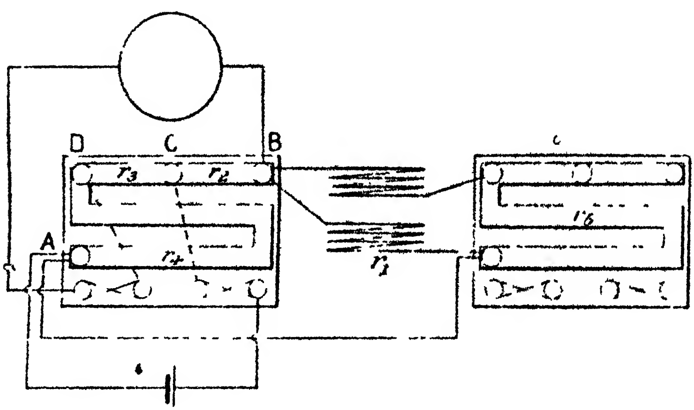
To compare the Mutual Inductance of Two Coils with the Self-inductance of one of them, and hence indirectly to determine it by Comparison with a Resistance.—One of the two coils is placed in the arm AB of a bridge, of which the remaining arms are non-inductive. The other coil is placed in series with another non-inductive resistance across two junctions of the arms, the resistance being adjustable.

The galvanometer is connected across the remaining two junctions, and the battery, having a key k in its circuit, to the

same points as the resistance and coil. A balance is then obtained for steady currents.

If now the battery key be opened or closed, a flow passes through the galvanometer, in general, owing to the inductance. This will not occur, however, if r_6 be suitably adjusted. The





adjustment should be made, and the resistance noted. The resistance, as put in by a box, plus that of the coil, which is determined later, is here denoted by r_6.

The bridge being now balanced for variable and steady currents, it follows that ($-$ L $\times$ current in AB) $=$ (M $\times$ current in r_6).

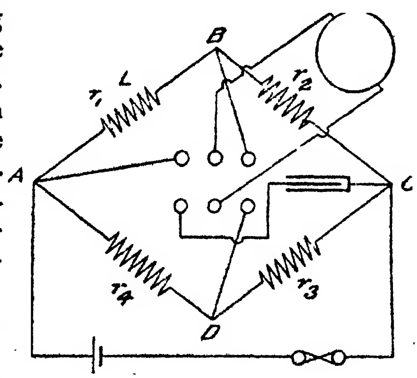
$$\text{Whence } M = -L\frac{r_5}{r_1 + r_2}$$

It is very essential that the least possible inductance, that of coils excepted, should exist in the bridge.

The value of L may be found by the method of Art. 142, and M determined in terms of resistances.

144.

To determine the Inductance of a Coil by Comparison with the Capacity of a Condenser.*—The coil of inductance L is placed in one arm of the bridge, the remaining arms of which are non-inductive. Three of the junctions between the arms are connected to a switch, the fourth being connected to the same through a condenser. By means of this switch either pair of alternate junctions may be connected through a galvanometer, the one pair directly, and the other through the condenser, each being open when the other is closed.



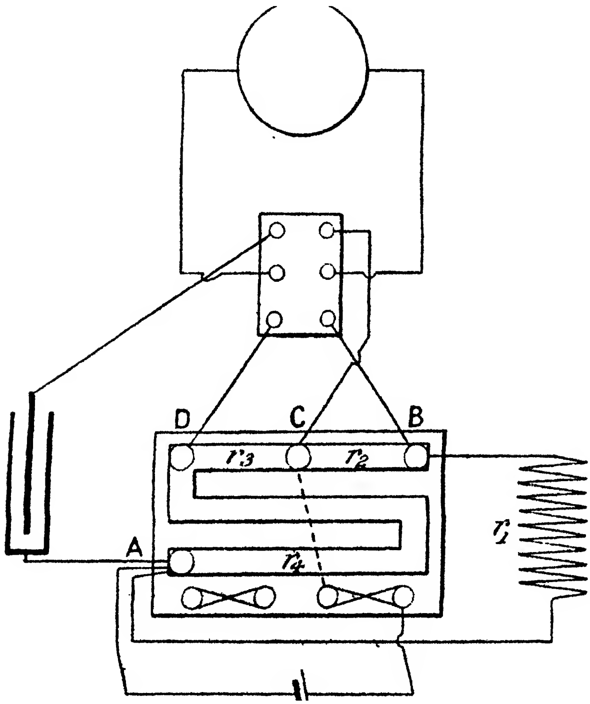
Let the coil be in AB, and a balance for steady currents obtained with BD closed.

Then if the key k be opened or closed, the inductance causes a flow through BD. Let the total amount of this be y, and the ballistic throw be θ_1.

If L′ and G denote the inductance and resistance of the galvanometer, $\frac{dx}{dt}$ the current at any instant in AB, $\frac{dy}{dt}$ that in

* Anderson, *Phil. Mag.*, April, 1891.

BD, and $\frac{dz}{dt}$ that in the battery, by Kirchoff's laws, in the circuits ABDA, BCDB—



$$L\frac{d^2x}{dt^2} + r_1\frac{dx}{dt} + L'\frac{d^2y}{dt^2} + G\frac{dy}{dt} - r_4\left(\frac{dz}{dt} - \frac{dx}{dt}\right) = 0$$

$$r_2\left(\frac{dx}{dt} - \frac{dy}{dt}\right) - r_3\left(\frac{dz}{dt} - \frac{dx}{dt} + \frac{dy}{dt}\right) - L'\frac{d^2y}{dt^2} - G\frac{dy}{dt} = 0$$

These integrated between $\frac{dx}{dt} = 0$ and $\frac{dx}{dt} = c$, the steady current in AB, give—

$$(r_1 + r_4)x + Gy = r_4 z - Lc$$
$$(r_2 + r_3)x - (r_2 + r_3 + G)y = r_3 z$$

since originally and finally $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$, and $\frac{d^2z}{dt^2} = 0$

$$\text{Hence } y = \frac{-Lc}{G\left(1 + \frac{r_1}{r_2}\right) + r_1\left(1 + \frac{r_3}{r_2}\right)}$$

The switch is then put over, connecting the condenser to A and C, and the ballistic throw θ_2 noted. If K be the capacity, θ_2 is due to a quantity of electricity $Kc(r_1 + r_2)$, $c(r_1 + r_2)$ being the difference of potential between A and C.

$$\text{Therefore } L = K(r_1 + r_2)\left\{G\left(1 + \frac{r_1}{r_2}\right) + r_1\left(1 + \frac{r_3}{r_2}\right)\right\}\frac{\sin\frac{1}{2}\theta_1}{\sin\frac{1}{2}\theta_2}$$

In practice it is convenient to adjust the capacity until θ_2 equals θ_1, or to find values of K giving θ_2 greater, and less, than θ_1, and to calculate the value that would give equality.

If L is small, r_1 and r_2 should also be small; if it be large, r_1 and r_2 should be greater. To increase r_1, a non-inductive resistance may be put in series with the coil in the arm AB. It is convenient to make r_1 equal r_2, and r_3 equal r_4, in which case, if θ_1 equal θ_2 also,

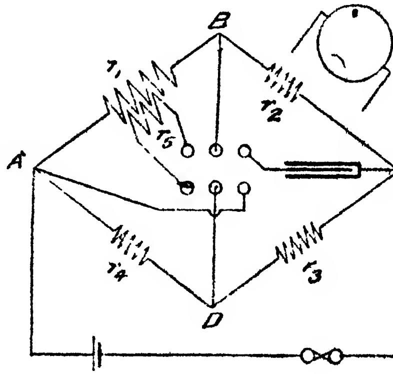
$$L = 2Kr_1(2G + r_1 + r_3)$$

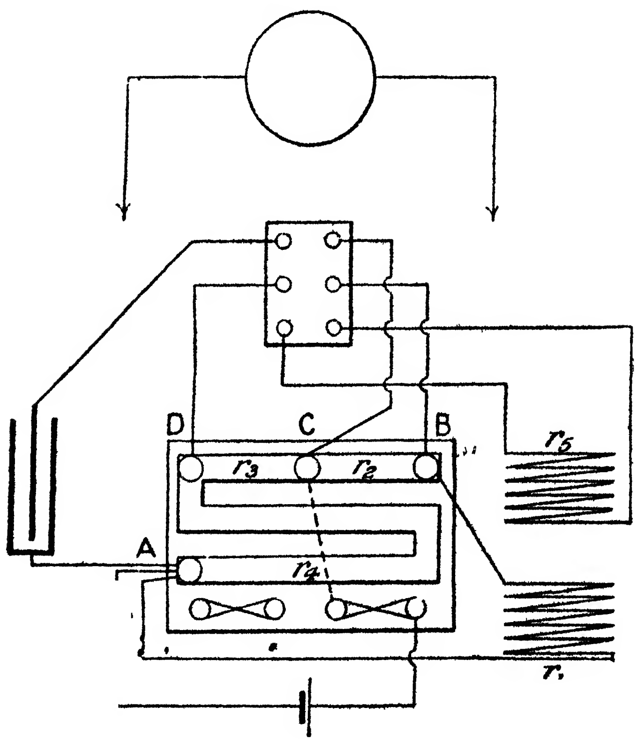
145.

To determine the Mutual Inductance of Two Coils by Comparison with the Capacity of a Condenser.—One of the mutually inductive coils is placed in one arm of a bridge, the remaining arms of which are non-inductive. The other is connected to mercury-cups so that it may be short-circuited through the galvanometer. The junctions of the arms are also connected to mercury-cups, one of them through the condenser.

Let the coil be in AB, and a balance for steady currents obtained with BD closed through the galvanometer. The inductive throw θ_1 is then observed when k is opened or

closed, also θ_2, due to the condenser charge when A and C are connected.





The throw θ_3 when the influenced coil r_5 is connected to the galvanometer, and k is opened or closed, is now noted.

The inductive electromotive force in this coil while the current is rising or falling is $M\frac{d^2x}{dt^2}$, if $\frac{dx}{dt}$ be the current in AB at any instant. This, integrated from $\frac{dx}{dt} = 0$ to $\frac{dx}{dt} = c$, the steady current in AB, is Mc. The whole quantity of electricity passing through the galvanometer is then $\frac{Mc}{r_5 + G}$, where r_5 is the resistance of the coil.

But (Art. 144) the whole quantity corresponding to θ_2 is $Kc(r_1 + r_2)$.

$$\text{Hence } M = K(r_1 + r_2)\,(r_5 + G)\frac{\sin \frac{1}{2}\theta_3}{\sin \frac{1}{2}\theta_2}$$

By determining M in this manner, and L as previously explained, the ratio of the numbers of turns in two coaxial coils may be found. The coil of the less radius must for this purpose be placed in the bridge arm.

146.

To determine the Permeability of a Specimen of Iron for Various Values of the Magnetizing Force by the Ballistic Method.—The magnetizing force, H, inside a solenoid having N turns per centimetre length is equal to $\frac{4\pi NC}{10}$, where C is the current in ampères. This is also the value of the induction, *i.e.* the number of magnetic lines per square centimetre of the sectional area, when the interior of the solenoid is non-magnetic.

If a magnetic core be introduced, the magnetizing force retains the same value, provided the length of the solenoid is not less than 400 diameters, but the induction is usually greatly increased. Let it be denoted by B.

The ratio $\frac{B}{H}$ is termed the permeability, and is usually denoted by μ.

The specimen of iron may be in the form of a straight rod, if of the proportions mentioned. Let a solenoid into which the rod just fits have N turns per centimetre length, and be connected through a resistance which can be abruptly varied by steps to an ammeter and battery. If another coil be wound round the central portion of the solenoid and connected to a ballistic galvanometer, a throw will be given when the number of lines in the solenoid is altered. Let the throw Θ, when a known current C is started or stopped in the solenoid, be noted before the iron is inserted.

After insertion, the bar is placed horizontally at right angles to the magnetic meridian, and demagnetized by a continually diminishing current sent through the solenoid in rapidly reversed directions.

A very small current c is now suddenly switched through the magnetizing coil, and the throw θ is noted. The increase of H is $\frac{4\pi Nc}{10}$, and the induction is $\frac{4\pi NC}{10} \cdot \frac{\theta}{\Theta}$.

For the next pair of readings, the current is suddenly increased to c' ampères, and the throw θ' observed. The value of H is thus $\frac{4\pi Nc'}{10}$, and the increase in B is $\frac{4\pi NC}{10} \cdot \frac{\theta'}{\Theta}$.

Proceeding thus by steps, the current should be increased to a maximum, reduced to zero, reversed, carried to a maximum, and ultimately brought to zero again.

Results should be tabulated thus:

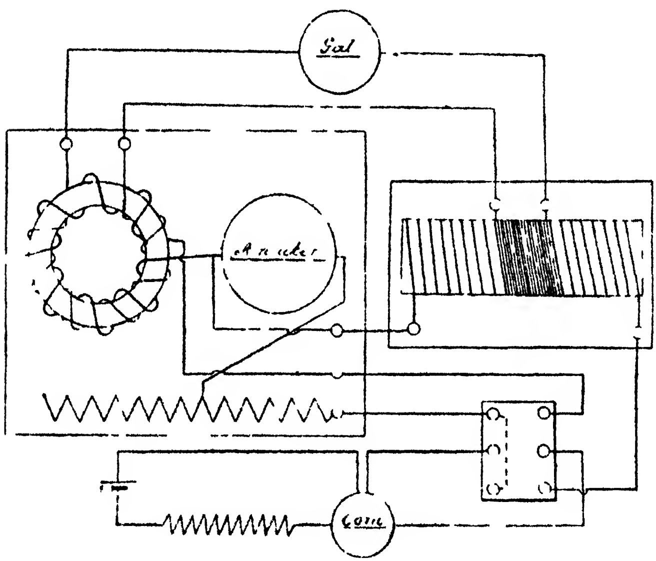
Ammeter reading.	Ballistic throw.	Change in B.	B.	H.	μ.

The values of B are obtained by adding together all the values of preceding lines in the column to its left.

A curve should then be plotted with values of H to a large

scale as abscissæ, and those of B to a much smaller scale as ordinates. The values of H are negative, and set off to the left after reversal; those of B are so when the deflection is reversed.

A much more satisfactory form of apparatus is obtained by making the iron into a radially-narrow ring and winding it with primary and secondary coils. A calibrating coil may have its primary and secondary similarly wound on a non-



magnetic core. The secondary of each coil should be connected to the other, and the circuit completed through the galvanometer. The primaries of the two coils should be connected to mercury-cups in such a way that the current may be sent through either.

It is sometimes an advantage to include an adjustable resistance in the primary circuit additionally to that by which the current is abruptly altered.

The latter may be a simple slab of paraffin wax with a row

This coil, like all the apparatus, must actually be so placed as to have no direct magnetic influence on any other piece.

of holes containing mercury, and a spiral of platinoid wire bridging each pair. Each resistance coil may then be cut out by dropping a short copper wire into its cups, until all are short-circuited.

The method of procedure is as previously explained. The values of B and H, however, are found as follows. Let the particulars of windings, etc., be given as below.

Iron Ring.		Non-magnetic Ring.	
Sectional area	$= \mathrm{A}$	Sectional area	$= a$
Primary turns	$= \mathrm{N}$	Primary turns	$= n$
Secondary turns	$= \mathrm{N}'$	Secondary turns	$= n'$
Mean circumference	$= \mathrm{L}$	Mean circumference	$= l$

Let θ be the ballistic throw given when a current, c, is started or stopped in the calibrating coil's primary, θ' that given by any sudden change in the current of the iron's primary. Then the change of induction in the iron core which produces this deflection is given by—

$$\text{Alteration in value of B} = \frac{4\pi n c a n'}{10\mathrm{AN}'l} \cdot \frac{\theta'}{\theta}$$

The magnetizing force in the iron ring with any current c' is $\dfrac{4\pi\mathrm{N}c'}{10\mathrm{L}}$.

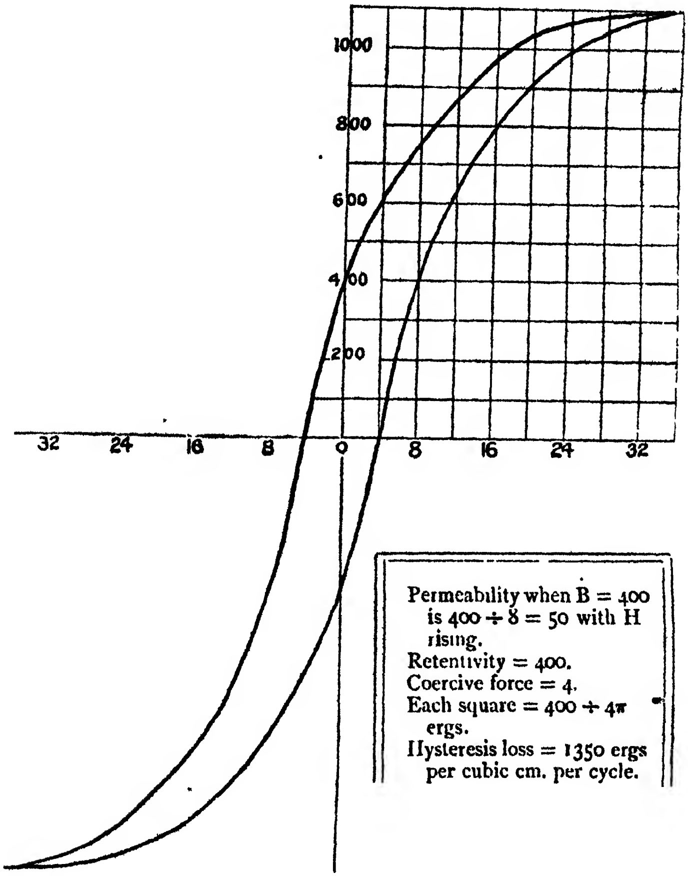
The results should be tabulated as shown above, and the curve plotted as explained.

The area enclosed by the curve when completely carried round and closed, divided by 4π, represents the loss of energy due to hysteresis in carrying unit volume of the iron through one cycle. This area being taken as a product of B and H, as plotted, gives the energy loss in ergs, when so divided.

The area included between the curve, the axis of ordinates, and the lines drawn through any two points on the curve parallel to the axis of abscissæ, divided by 4π, represents the energy expended in increasing the induction between those limits. This remark refers only to the curve of increase of

induction; on that of diminishing induction, it represents the energy given up by the iron.

The point at which a tangent drawn from the origin to the curve, touches the latter, is that at which the expenditure of



Permeability when B = 400 is 400 ÷ 8 = 50 with H rising.
Retentivity = 400.
Coercive force = 4.
Each square = 400 ÷ 4π ergs.
Hysteresis loss = 1350 ergs per cubic cm. per cycle.

energy in producing induction, and the electro-kinetic energy of the magnetizing circuit, are increasing at the same rate. The latter is equal at any point to $\frac{1}{8\pi}$HB.

The induction remaining when a diminishing current reaches zero is termed the "retentivity," and that value of H which brings a diminishing induction to zero the "coercive force."

The following should therefore also be stated:—

Hysteresis loss in ergs per cycle =
Rates of loss of energy in producing B and of increase of energy in circuit equal at
Retentivity =
Coercive force =

147.

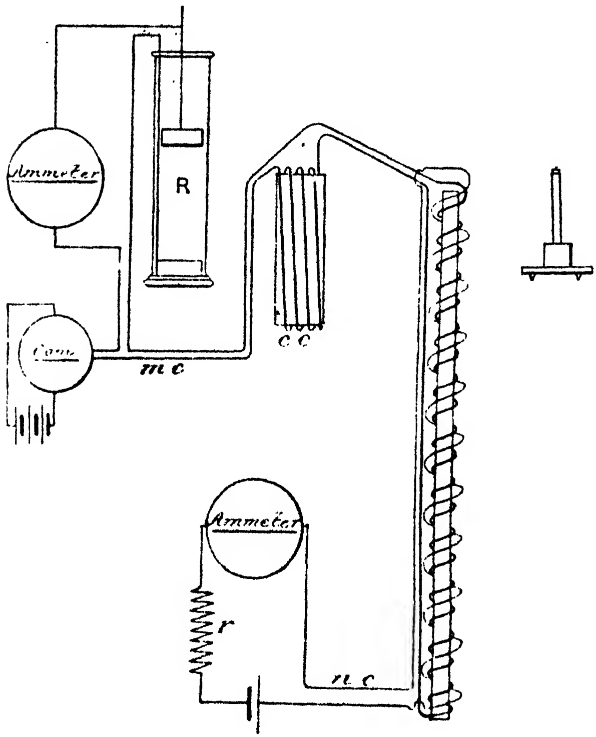
To determine the Permeability of a Specimen of Iron for Various Values of the Magnetizing Force by the Magnetometric Method.—In this method a bar of the iron not less than four hundred diameters long is employed. This is placed in a magnetizing solenoid which extends some little distance beyond its own ends, and supported vertically in a plane at right angles to the magnetic meridian and passing through the centre of the magnetometer needle.

To neutralize the effect of the solenoid's poles, a compensating coil is placed vertically in such a position that its axis produced passes through the magnetometer needle, and is at right angles to the magnetic meridian, its distance from which is adjusted till no effect is produced on it when a current passes through the coil and solenoid in series. The neutralization is then complete for all values of the current.

Around the solenoid should be wound a second coil of fewer turns for the purpose of counteracting the vertical component of the earth's magnetic force. This should be connected to a cell of constant electromotive force through an adjustable resistance *r*. All out and return wires must be closely side by side so as to be non-inductive.

The magnetizing solenoid and compensating coil should be connected to their battery through a commutator, a variable resistance, and an ammeter.

This resistance may be conveniently made by suspending zinc cylinders connected to insulated wire in a tall vessel of saturated zinc sulphate, as shown at R. The current may then be adjusted by altering the distances the cylinders are apart, for which purpose the upper one may be suspended.



The rod must now be adjusted, with its solenoid, vertically until, with a constant current in *mc*, the maximum effect is produced on the needle. In this position its resultant pole is level with the latter.

The next operation is to neutralize the earth's force. To do this a small current is sent through the circuit *nc*, and the iron bar inserted in the solenoid. This, when the current in *nc* is duly adjusted, will be completely demagnetized by sending

a strong current through the magnetizing solenoid and gradually reducing it to zero, meanwhile reversing it rapidly by means of the commutator. This must be done for various values of current in *nc*, until no effect is produced by the rod on the magnetometer.

It is as well now to note whether the empty solenoid and compensating coil still balance, and to adjust further, if necessary, the latter's position.

The various parts being now adjusted and clamped in position, the observation of current in *mc*, and of deflections, commences.

The current is increased by noted steps to its maximum value, and the magnetometer deflection for each noted.

The same observations are made, reducing the current to zero, reversing, carrying to a maximum again, and finally bringing to zero, after which it should again be carried to a maximum in its original direction.

Results should be tabulated thus:

Current.	Magnetometer deflection.	H.	I.	B.	μ.

$H = \frac{4\pi NC}{10}$, where N = turns per centimetre in magnetizing solenoid, C = current in ampères. This H is not, however, the true value of the total magnetizing force in the solenoid, for the poles of the iron produce, themselves, a reverse force of amount LI, where L is a number depending on the ratio of the rod's length to its diameter, and for which Professor Ewing gives the following values:

$\frac{\text{Length}}{\text{diameter}}$	L.	$\frac{L}{4\pi}$
50	0·01817	0·001450
00	0·00540	0·000430
6o	0·00157	0·000125
00	0·00075	0·00006o
400	0·00045	0·000037

Hence H′, the operative force, $= \mathrm{H} - \mathrm{LI} = \mathrm{H} - \frac{\mathrm{LB}}{4\pi}$ approximately, if B be large compared with H′.

I, the intensity of magnetization, is deduced from the deflection θ of the magnetometer as follows :—

$$\text{Deflecting force} = \mathrm{F} \tan\theta = \frac{\pi a^2 \mathrm{I}}{d_1^2} - \frac{\pi a^2 \mathrm{I}}{d_2^2} \cdot \frac{d_1}{d_2} = \frac{\pi a^2 \mathrm{I}}{d_1^2}\left(1 - \frac{d_1^3}{d_2^3}\right)$$

$$\text{Hence } \mathrm{I} = \frac{d_1^2 \mathrm{F} \tan\theta}{\pi a^2\left(1 - \frac{d_1^3}{d_2^3}\right)} = \left\{\frac{d_1^2 \mathrm{F}}{2\pi a^2 l\left(1 - \frac{d_1^3}{d_2^3}\right)}\right\} \times \left\{\begin{array}{c}\text{scale reading} \\ \text{in cms.}\end{array}\right.$$

where F = the horizontal component of the earth's magnetic force, a = radius of the rod, d_1 = distance between centre of needle and rod horizontally, d_2 = distance between centre of needle and a corresponding point on the rod near its lower end, l = the distance between the centre of the needle and the zero of the scale.

B, the induction, $= \mathrm{H} + 4\pi\mathrm{I}$ very approximately.

The determination of μ, the permeability, is better postponed until a curve has been plotted in the following manner :—

Axes are drawn in the usual way, and a line is drawn from the origin, making with the axis of ordinates an angle $\tan^{-1}\frac{\mathrm{L}}{4\pi}$ measured to the left. Values of B are then plotted as ordinates, at distances from this line proportional to the corresponding values of H. The true relation between B, the induction, and H′, the actual magnetizing force, is then given by the distances from the rectangular axes as usual. For negative values of H, the oblique line is produced downwards, and lies to the right of the axis. The permeability is then $\frac{\mathrm{B}}{\mathrm{H'}}$.

There is, however, no difficulty in finding H′ from the relation given above, and B $(= \mathrm{H'} + 4\pi\mathrm{I})$, and so plotting the curve directly and more accurately.

The expenditure of energy on unit volume of the iron per cycle is given in ergs by the area enclosed by the curve divided by 4π.

The determination may also be made by placing the bar horizontally, level with the needle, in an easterly and westerly direction, its centre being due north or south of the centre of the needle. In this case the earth's field does not affect it, and the neutralizing coil is unnecessary.

The poles should be regarded as lying one-sixth of the bar's length from each end.

148.

To determine the Characteristic Curve of a Dynamo.—The characteristic curve for any speed shows the relation between the electromotive force induced in the armature and the current flowing through it at that speed.

The resistance of the shunt coils, and of the armature with the series coils, should be separately found.

A voltmeter is then connected across the terminals, and an ammeter put in series with variable resistances through which the current is to be sent.

It is very convenient to drive the dynamo by a motor, itself driven by a current from mains at a constant potential. In this case an adjustable resistance is also required in the motor circuit, so that the speed of the dynamo may be kept constant while its current is varied.

The maximum resistance being put in the motor circuit, the dynamo is run on open circuit, *i.e.* with the terminals only connected through the voltmeter. If the speed, which is now observed, be greater than that for which the characteristic is required, it is necessary to place even greater resistance in the motor circuit, or to apply some form of brake. If it be lower than required, it may be adjusted by reducing the motor resistance. At the required speed the voltmeter is read.

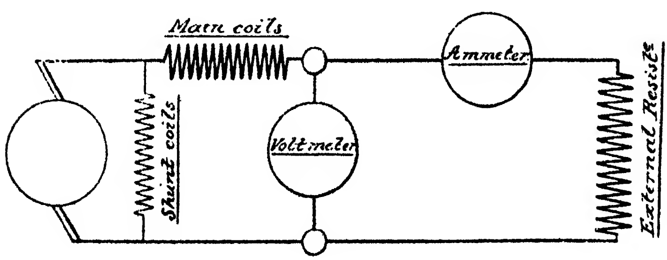
The external resistance of the dynamo is now made as great as possible, for which purpose a resistance formed by two leaden electrodes and very dilute acid is convenient, and, the motor having been stopped, it is connected to the terminals *a* and *b*. The machine is then run and its speed read. The

resistance in the motor circuit must be reduced till the speed is as before, when the voltmeter and ammeter are read. The external resistance is then further reduced, and also that of the motor, to keep the speed up.

Proceeding in this way, reducing the external resistance, reading the speed and adjusting the motor resistance, then reading the instruments, a series of points connecting the external voltage and current are obtained.

These operations necessarily cease when all resistance is taken out of the motor circuit, or when the dynamo is carrying its maximum current.

The latter limit is inevitable, but observations may be carried beyond the former by determining the connection between speed and E.M.F. at the terminals. The observed occurrence upon reducing the external resistance, after all in



the motor circuit has been taken out, is that the speed falls below its required value, with a consequent drop in the voltage.

The voltage corresponding to the characteristic's speed may be deduced as follows :—

The external resistance is reduced till the required current flows, and the speed and E.M.F. are read. Let these be N_1 and V_1.

The speed is then altered by means of the motor resistance, the external resistance adjusted till the current has its previous value, and speed and E.M.F. again read. Let these now be N_2 and V_2.

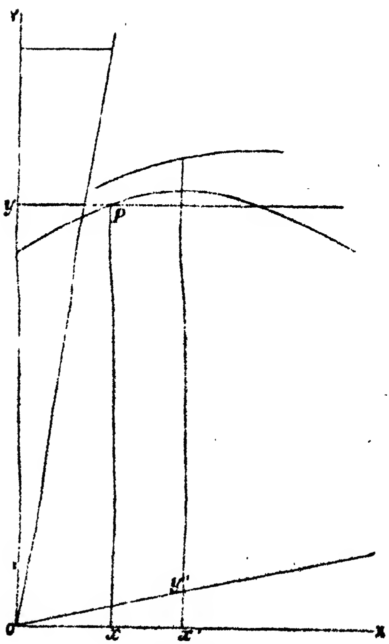
If V be the voltage, with the existing current, at the correct speed N—

$$\frac{V - V_2}{N - N_2} = \frac{V_1 - V_2}{N_1 - N_2}$$

$$V = \frac{N - N_2}{N_1 - N_2}(V_1 - V_2) + V_2$$

Thus observations may be made, and points on the curve found, for all currents between zero and the maximum.

A curve should then be plotted with values of the external current as abscissæ and of the external electromotive force as ordinates. This is the external characteristic.



To construct from this the characteristic curve, let r be the resistance of the shunt (it will be more accurate to increase the resistance as measured when cold by 3 per cent.), and let a point, $x = 1, y = r$, be joined to the origin. From any point p on the curve, let lines px, py be drawn parallel to the axes. Then the abscissæ of the point at which py cuts the inclined line gives the current in the shunt coils when px is the external voltage. Let x' be set off from x at a distance equal to this abscissa.

To correct for the drop in E.M.F. in the armature and series coils (which should have their resistance increased by about 3 per cent. if measured cold), a line should be drawn from the origin, making with OX an angle whose tangent is

numerically equal to their resistance. Let an ordinate be drawn from x', cutting this line at y'.

The total induced E.M.F. in the armature, when the armature current is Ox', is then equal to $px + x'y'$, and the point corresponding to this E.M.F. is set off on the ordinate through x'.

The other points on the curve are derived from the external characteristic in a similar manner.

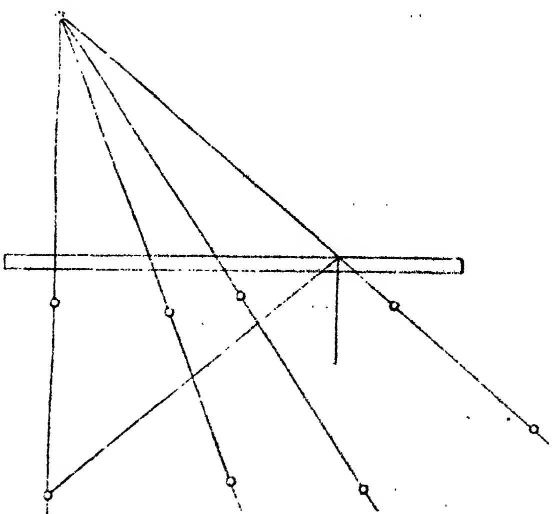
If the dynamo be series wound, no correction is, of course, necessary for shunt current.

PART IV

LIGHT AND OPTICS

149.

To determine the Position of an Image seen in a Plane Mirror, and to prove that the Angles of Incidence and Reflection are Equal.—A plane mirror is supported in a



vertical plane, on a sheet of paper, and a line drawn on the latter marking the position of the reflecting face.

The most convenient image to deal with is that of a pin

driven through the paper, and standing vertically some inches in front of the mirror near one end.

The eye being then placed so that the pin and its image coincide, another pin should be fixed in line with them. The line which may be drawn later through these two pinholes is then at right angles to the reflecting-plane.

The eye is now moved sideways until the first pin is seen reflected in some new position. Two more pins are then placed in the same line with the eye, so that they and the image coincide.

This operation is repeated for different positions, after which the mirror may be removed, and straight lines ruled through the different pairs of pinholes to intersect the mirror line and meet behind it.

It will now be found that the image is on that normal to the mirror which passes through the object, and that it is situated at the same distance behind the surface as the object is in front of it.

By drawing straight lines from the first pin to the points in which those previously drawn intersect the mirror line, and drawing perpendiculars at these points to the mirror line, it may be shown that the angles of incidence and reflection are equal.

150.

To determine by Means of a Sextant the Angle subtended at the Eye of the Observer by Two Distant Objects.—The ordinary form, known as Hadley's sextant, consists of a triangular frame, of which one side is an arc having its centre at the intersection of the other two. One of the straight arms carries a telescope directed towards a glass plate on the other. The upper half of this glass plate is silvered so that one-half the field receives transmitted, the other half reflected, rays. Movable about the centre of the arc is a third arm, carrying a mirror so adjusted that when this arm coincides in position with that carrying the telescope, the latter receives rays,

travelling parallel to its axis before incidence, by successive reflection from both mirrors.

If the direction of the rays before incidence makes an angle θ with the axis of the telescope, it is necessary that the arm should be turned through an angle $\frac{\theta}{2}$, in order that they may pass through the latter.

Hence, if the instrument be held so that some fixed point is seen directly by transmission through the half-mirror, close to the dividing line between its halves, and the movable arm be turned until the image of some other point coincides with it, the angular distance between the two points is twice the angle between the movable and telescope arms.

To enable this reading to be made directly, the graduations on the arc are numbered with twice their real value, and the movable arm carries a vernier.

Darkened glasses are usually provided, in order that the sun's position may be found.

The zero error of the instrument should be determined by turning the arm until the direct and reflected images of some point coincide, and reading its position on the arc. If this position be outside the telescope arm the zero error is added; if inside, it is deducted from the readings made subsequently.

In measuring the altitude of a star or of the sun, the horizon is viewed directly, and the arm turned until the sun's lower edge coincides with it. Under many circumstances the horizon is not visible, and then the telescope is usually directed towards the image of the sun in a dish of mercury. When adjusted so that the images just touch each other, the reading is double the altitude from the horizon.

151.

To determine the Radius of Curvature of a Concave Mirror.—The following method, based upon the fact of coincidence between object and image at the centre of curvature, is applicable, and usually the most convenient:—

A needle, or other object with a pointed end, is clipped vertically in a stand, its point being approximately level with the centre of the mirror.

Its distance from the latter is then adjusted until no relative motion occurs between the image and itself when the eye is moved from side to side.

The vertical position is now altered until the points of the needle and its inverted image just meet, and the previous adjustment again made, more exactly.

To facilitate this, a lens may be focussed upon the needle-point.

The distance between the needle-point and the mirror is then the latter's radius of curvature.

If the mirror be large, it may be necessary to screen off part, and to use only the central portion, to obtain a distinct image. This may be done by placing a sheet of card with a circular hole in front of the mirror.

The measurement of the radius from the point to the surface may be made directly by a scale for a large mirror, or by means of compasses if it be small.

Methods (*b*) and (*c*) of the next article may also be applied for this determination.

152.

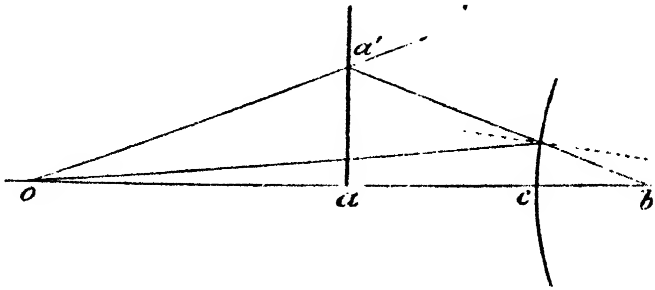
To determine the Radius of Curvature of a Convex Mirror.—(*a*) The mirror is supported on a stand, and receives rays diverging from a narrow slit in a screen placed before it, or from brightly illuminated cross-wires.

The rays, after reflection, diverge from a virtual image situated behind the mirror.

If now a sheet of clear glass be interposed between the slit or cross-wires and mirror, the rays are again reflected, diverging from a second virtual image as far from the front face of the glass as the first virtual image is from its back face.

The position of the glass sheet is adjusted until the object

and this final image coincide without parallax when viewed through the glass. In this case, if o be the object, aa' the



glass, c the surface of the mirror, and b the position of the image in the mirror—

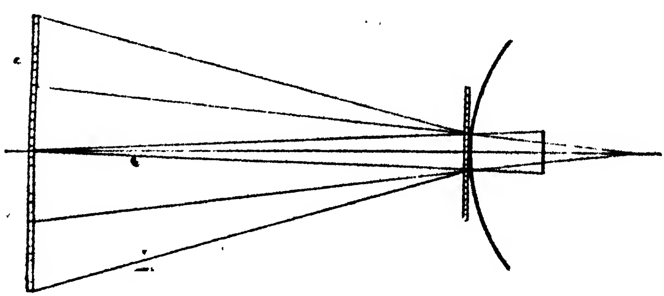
$$oa = ab$$
$$oa - ac = cb$$

Hence, by measuring oa and ac, the radius, R, may be found from the ordinary relation between conjugate foci.

$$\frac{2}{R} = \frac{1}{oa - ac} - \frac{1}{oa + ac} = \frac{2ac}{oa^2 - ac^2}$$

$$R = \frac{oa^2 - ac^2}{ac}$$

(*b*) If a scale be placed before the mirror so that its centre



is on the latter's axis and its length is perpendicular to it, a virtual image is produced.

The eye being placed at the centre of the scale, the length of the image as given by a scale placed parallel to the first and in contact with the mirror is observed.

Let the distance between the surface of the mirror and the distant scale be L, the length of this scale be D, and the apparent length of the image as seen against the other scale be d.

Then, by similar triangles—

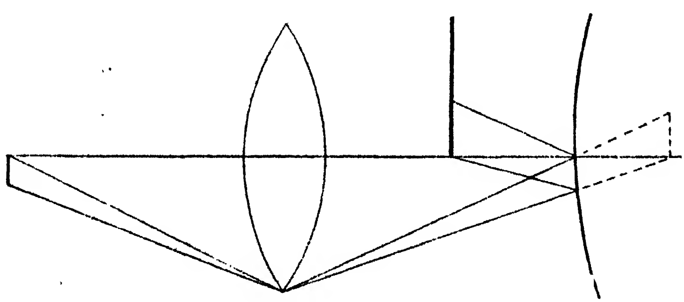
$$\frac{\frac{1}{2}D}{L+R} = \frac{d}{R} \text{ approximately}$$

$$\frac{\frac{1}{2}D - d}{L} = \frac{d}{R}$$

Whence the radius, R, is given by—

$$R = \frac{Ld}{\frac{1}{2}D - d}$$

(*c*) The determination may also be made by the use of an accessory convex lens of short focal length.



The lens, screen, and illuminated cross-wires being set up, a real image of the latter is obtained, and the position of the screen noted.

The screen is then removed, and the reflecting surface substituted at a less distance from the lens.

If the rays are sufficiently convergent and near the axis, a

real image is now formed in front of the mirror. The position of this may be found and noted by placing a piece of card across part of the reflected rays.

Let the difference of the distances of the screen and reflecting surface from the lens be D, and the distance of the reflected image from the mirror be d.

$$\text{Then } \frac{2}{R} = \frac{1}{d} - \frac{1}{D}$$

$$R = \frac{2Dd}{D - d}$$

153.

To determine the Index of Refraction of a Solid.—The refractive index of a solid, usually denoted by μ, is the ratio of the velocity of light in air to its velocity in the substance.

This equals the ratio of the sine of the angle of incidence to that of refraction when a ray passes from air into the solid.

The absolute index is the ratio of the velocity *in vacuo* to that in the substance.

The index relatively to air may be determined by several methods.

(*a*) Where the substance is transparent, and a block with two parallel faces is available, the following simple method may be used:—

A sheet of paper being laid upon a board, the block is placed on it with the parallel faces vertical.

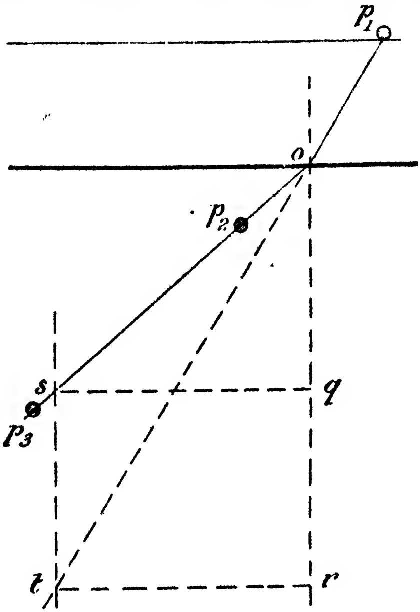
The positions of these are marked on the paper, and a pin is then driven into it, in contact with the back face and near one end of it.

This pin, marked as p_1, is viewed through the block, and the eye moved to one side so that the line of vision is oblique to the face. Two pins, p_2 and p_3, are then inserted some distance apart in line with the eye and the first pin.

The block may now be removed, and a straight line drawn through p_2 and p_3, cutting the front face line in some point o.

Another line through p_1 and o should be produced considerably beyond o.

The refractive index is then equal to the ratio of the



lengths of these lines, measured from o, cut off by any straight line perpendicular to the face line.

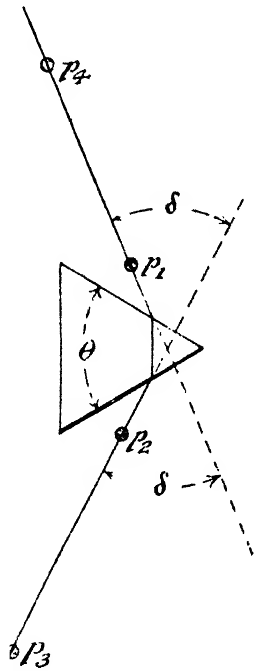
$$\text{For } \frac{\sin i}{\sin r} = \frac{\frac{qs}{os}}{\frac{rt}{ot}} = \mu$$

$$\text{and } qs = rt$$

$$\text{therefore } \mu = \frac{ot}{os}$$

(*b*) If the substance be in the form of a prism, the refractive index may be found by placing it with its refracting edge vertical upon a sheet of paper, inserting a pin some distance behind it, and turning until the image seen through it is as

near as possible to the actual pin. If the prism be turned about a vertical axis, it will be observed that, rotated in one direction, the image and object approach until a limiting position is reached, when the direction of the image's motion is reversed.



This position, termed that of minimum deviation, being obtained, two pins, p_2 and p_3, are inserted some distance apart in line with the eye and the image of p_1.

There is placed also behind the prism a pin, p_4, in such a position that it appears, when seen through the prism, to be in line with p_1, p_2, and p_3.

The prism is now removed, and straight lines drawn through p_1 and p_4, p_2 and p_3. The angle δ between these is the angle of minimum deviation.

It may be proved that in this position the angles of incidence and emergence are equal, and that the angles of refraction and incidence inside the substance are together equal to θ, the refracting angle, and equal to each other.

$$\text{Hence } \delta = 2i - \theta, \text{ or } i = \frac{\delta + \theta}{2}$$

$$\text{and } \theta = 2r, \text{ or } r = \frac{\theta}{2}$$

$$\text{But } \sin i = \mu \sin r$$

$$\text{therefore } \sin \frac{\delta + \theta}{2} = \mu \sin \frac{\theta}{2}$$

$$\text{and } \mu = \frac{\sin \frac{1}{2}(\delta + \theta)}{\sin \frac{\theta}{2}}$$

whence, θ being measured, μ may be calculated.

(*c*) The determination by (*b*) may be more satisfactorily made by means of a spectrometer.

The spectrometer consists of a telescope, which may be rotated about a vertical axis, and for which the angular position is indicated by two verniers travelling over a graduated ring.

The arm carrying this telescope is fitted with a radial clamping-screw and a tangent screw for making exact adjustment, while the telescope itself has a Ramsden eye-piece and cross-wires.

At one point of the graduated ring is permanently fixed a collimating tube in the same horizontal plane as the telescope. This has at its outer end a narrow slit of adjustable width, which is so placed as to be at the principal focus of a lens fitted at the inner end of the tube.

One-half of the slit is often capable of having a totally reflecting prism turned in front of it. In this way the light from two sources may be compared, that from one passing directly in, that of the other reflected in by the prism at right angles to its original direction.

Coaxially with the graduated ring is usually fitted a central table, sometimes capable of rotation, and having its position indicated by verniers.

To adjust the instrument before a determination, the eye-piece of the telescope should be focussed on the cross-wires, and, without relative motion, both moved until some very distant object is seen without parallax, *i.e.* with no motion relatively to the cross-wires when the eye is moved about. The telescope is then focussed for parallel rays.

It is now turned so as to look directly into the collimator tube, and the distance between the slit and its lens is adjusted until the former is in focus.

The telescope is then clamped, the vertical cross-wire brought to the centre of the slit by means of the tangent screw, and the verniers read.

The prism being placed on the table so that the light from the collimator passes through it, and, with its refracting edge vertical, the telescope is turned until the image of the slit is

again seen. To place the prism in the position of minimum deviation, it is rotated so as to bring the image as nearly as possible to its original position. During this rotation, the telescope also is turned so as to keep the image in view, and upon the limiting position being attained, it is clamped, and the final adjustment made by moving the prism slowly and turning the cross-wire to the centre of the image.

Both verniers are now again read.

The prism should be turned so that the light is incident upon the other face and the deviation in the opposite direction, the above operation being repeated.

Let the mean angle of deviation be δ.

It is necessary that the angle of the prism should now be found. This may be done by setting the prism with its refracting edge toward the collimator and slightly in front of the centre of the table. The light from the slit will then be thrown off to right and left by reflection from the faces of the prism. If the telescope be turned to receive it from each face in succession, and the verniers be read for each position, the angle between the two positions is double that of the prism itself.

If the edge of the prism be not sharp, the following method may be used. The telescope is clamped in any position less than 180° with the collimator, and the prism-table turned until an image of the slit is seen reflected from one face. The verniers indicating the position of the table are thereupon noted. The prism-table is then turned until the image is seen reflected from the other face, when the verniers are again read.

The direction of the table's rotation is such that the two sides containing the angle reflect in succession.

In this case the angle turned through by the table equals 180° minus the angle between the faces.

Let θ be the angle of the prism.

Then, as shown above—

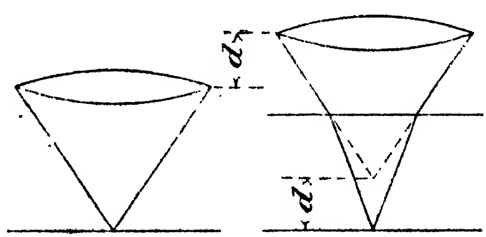
$$\mu = \frac{\sin \frac{1}{2}(\delta + \theta)}{\sin \frac{\theta}{2}}$$

The light, which should be placed as near to the collimator slit as possible without heating the instrument, must be monochromatic, as otherwise the width of the slit is drawn out by the prism into a spectrum band.

On this account a sodium flame is generally used. A simple Bunsen burner, with a piece of platinum foil, or even iron wire, carrying a quantity of sodium carbonate, in its flame, is fairly satisfactory. It should be protected from draughts by a chimney, as otherwise the flickering is very objectionable. It is desirable to have a brighter light which may be switched on for reading the verniers near at hand.

(*d*) The index of refraction of a body, even when opaque, may be determined by finding its polarizing angle. For, as discovered by Brewster, the tangent of this angle is numerically equal to the refractive index. The details of this method are dealt with in Art. 170.

(*e*) If the solid be transparent and in the form of a plate, its index of refraction may be found by means of a micro-



scope. For this purpose a pointer should be rigidly attached in a horizontal position to the sliding-tube carrying the lenses, and a finely graduated scale fixed parallel to the axis of the instrument. This pointer will indicate the distances the tube is screwed up or down.

The tube of the microscope should be turned to a vertical position, and a piece of card having two intersecting lines ruled upon it placed upon its stage.

The microscope is adjusted until the point of intersection of the lines is in focus, and the reading of the pointer noted.

The slab of the substance is now placed over the card, the instrument again focussed, and the scale reading noted.

Let the difference of these readings be d.

The thickness of the plate is now found by a micrometer gauge, or by focussing the microscope on a mark on its upper surface and reading the scale, in which case the difference between this and the first reading is its thickness.

Let this be D.

$$\text{Then } D = \mu(D - d)$$

$$\mu = \frac{D}{D - d}$$

154.

To determine the Index of Refraction of a Liquid.—(*a*) The simplest method of determination is to employ a hollow glass prism and proceed as in method (*c*) of the last article. If the sides of the prism be of uniform thickness, the whole deviation is due to the liquid inside.

(*b*) The refractive index may also be found by observing the polarizing angle as explained in Art. **170**.

(*c*) In applying the microscope method, a mark should be made upon the bottom of a vessel, and about a centimetre's depth of the liquid poured in. The microscope is focussed firstly upon the mark at the bottom, and then upon some light powder floating on the surface of the liquid.

Let the difference between the observed scale readings for these two positions be d_1.

Then, if d be the actual depth of the liquid—

$$d_1 = \frac{d}{\mu}$$

A further quantity of liquid is poured into the vessel, increasing its depth by some amount D.

The previous observations are now repeated, and the difference of the scale readings noted.

Let this be d_2.

$$\text{Then } d_2 = \frac{d + D}{\mu}$$

$$\text{Therefore } d_2 - d_1 = \frac{D}{\mu}$$

$$\mu = \frac{D}{d_2 - d_1}$$

Results should be stated thus:

(1) Focus on bottom mark. Scale reading =
(2) " surface " " =
(3) " bottom mark after adding liquid. Scale reading =
(4) Focus on surface mark after adding liquid. Scale reading =

d_1 = (2) minus (1) =
d_2 = (4) minus (3) =
D = (4) minus (2) =
μ =

(*d*) Since the critical angle when light passes from a medium of refractive index μ into air satisfies the equation—

$$\sin i = \frac{1}{\mu}$$

it is possible to determine μ by finding the critical angle.

The most convenient apparatus for this purpose is a box with parallel glass windows in two opposite sides Attached to a vertical axis in the centre of the box are a pair of thin glass plates cemented together round their edges, and containing a film of air between them. An index fixed to the axis indicates on a graduated circle above the box the angular position of the plates.

The spectrometer being adjusted, as explained in the last article, with a sodium flame behind the slit, the telescope is turned to view the latter directly.

The box, filled with the liquid, is placed on the table, the plates turned round so that the slit is seen through the windows, and the position of the box adjusted until the slit is seen in its original position.

The plates are now placed so that their plane is perpendicular to the axis of the telescope. By turning them in either direction a position is found at which the light is extinguished owing to total reflection. The angle between these two positions of extinction is double the critical angle; and hence μ may be calculated by the above expression.

155.

To determine the Focal Length of a Convex Lens.—The focal length of a lens is the distance between the point toward or from which parallel incident rays converge or diverge after passing through the lens and another point termed the principal point. This last point when the lens is thin is generally assumed to be at the centre of its thickness.

The following methods may be employed to find this length when the lens is convex:—

(*a*) A paper screen is fixed in one stand of an optical bench, and the lens in another in front of it. The lens and screen are turned toward a window through which some distant object is visible, and the distance between them varied until a distinct image is formed on the screen. It is necessary to prevent the screen from receiving more general light than possible. The distance between the centre of the lens and the screen, which is very approximately the focal length, may now be measured or observed from the bench graduations.

If the distinctness of the image does not perceptibly vary for small displacements of the lens, the two limiting positions of distinctness should be noted, and the mean value taken as the focal length.

(*b*) A lamp is placed at one end of the bench, and upon three stands are placed cross-wires, the lens, and a paper screen. The cross-wires conveniently consist of a card with

a circular hole having two threads stretched across it. This is placed between the lamp and the lens, the screen being behind the latter. The centres of the flame, cross-wires, and lens being adjusted in one horizontal line, the cross-wires and screen are separated to a distance greater than four times the focal length.

The lens is now moved until a distinct image of the cross-wires is formed on the screen, and its distances from the cross-wires and screen noted. The sum of the reciprocals of these distances is equal to the reciprocal of the focal length. Observations should be made with the cross-wires and screen at different distances apart, and the mean value of the focal length found.

(*c*) The apparatus of method (*b*) being employed, and the distance between the cross-wires and screen being greater than four times the focal length, two positions exist, in either of which the lens gives a distinct image.

These two positions are found, and by diminishing the distance between the screen and cross-wires are made to approach and coincide.

In this condition the focal length is one-fourth of the distance between the cross-wires and screen.

The observation may be made with greater accuracy if a lens focussed upon a needle-point be used in place of the screen.

When the image of the cross-wires coincides in position with the needle-point, both are seen through this lens without parallax.

As no measurements from the surfaces of the lens are necessary, this method is particularly applicable in the case of those that are thick, and of combinations.

In accurate work, however, it is better to find the principal points and focal lengths by the focometer.

156.

To determine the Focal Length of a Concave Lens. Owing to the image given by a concave lens being virtual, the methods applied in the case of convex lenses are not available without an auxiliary convex lens of less focal length.

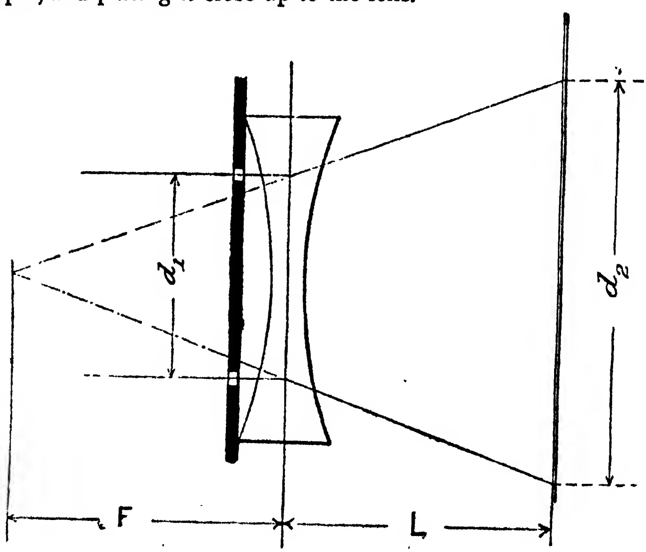
(*a*) If the concave lens be placed close up to a convex one having a focal length less than its own, the combination may be regarded as a convex lens, and its focal length found as such.

If F be the focal length of the concave lens, f_1 that of the combination, and f_2 that of the convex lens—

$$\text{then } \frac{1}{F} = \left(\frac{1}{f_2} - \frac{1}{f_1}\right)$$

$$F = \frac{f_1 f_2}{f_1 - f_2}$$

(*b*) The focal length of a concave lens may be determined approximately by cutting two narrow parallel slits in a sheet of paper, and placing it close up to the lens.



If parallel rays pass through these slits, they diverge after passing through the lens, and their image may be received upon a darkened screen.

Let d_1 be the distance apart of the slits in the paper, d_2 the distance apart of their images. As the images are blurred,

these distances should be measured to the centres of width. Let L be the distance between lens and screen, F the focal length of the lens.

Then, by similar triangles—

$$\frac{F}{L} = \frac{d_1}{d_2 - d_1}$$

or—

$$F = \frac{d_1 L}{d_2 - d_1}$$

157.

To determine the Positions of the Primary and Secondary Focal Lines for Light obliquely Incident upon a Mirror or Lens.—Light diverging from a point and passing obliquely through a lens is thereby caused to converge, not to a single point, but to two lines lying in perpendicular planes and at different distances from the lens. A similar effect is, under the same circumstances, produced in the rays reflected from a mirror.

The lens or mirror should be carried on a turn-table fitted to one upright of an optical bench, the turn-table itself having a graduated circle whereby the angle between the axis of the lens or mirror and that of the bench may be measured.

If illuminated cross-wires be employed as the object, a distinct image of the vertical wire appears at one position of the screen, and that of the horizontal wire at another. These positions are those of the primary and secondary focal lines respectively.

The object being placed at some fixed distance from the lens or mirror, the axis of the lens is made to coincide with that of the bench, and the position of the focal lines, which now coincide, is found. The lens or mirror is now turned through some small angle, and the positions again noted. The same observations are repeated for as many angles as possible between 0° and 90°.

The distance between the object and the lens or mirror should now be altered considerably, and the whole of the previous observations repeated.

Denoting by D the distance between the cross-wires and the centre of the lens or mirror, by d_1, d_2 that between the lens and the primary and secondary focal lines respectively, the observations should be tabulated thus:

Angle.	D.	d_1.	d_2.

The positions should be plotted by drawing a straight line to denote the axis of the lens or mirror, and through a point upon it drawing a number of other straight lines, making angles with the first equal to those the axis made in the actual observations. This angle is marked upon each line, and the lengths d_1, d_2 are set off from the common point of intersection. From the same point the lengths D are set off in the reverse direction. Curves are now drawn through the primary and secondary loci corresponding to a particular value D, and also through the points indicating the latter. Each set of curves thus consists of three, and the different sets should be drawn with lines of distinctive character. The lengths may, of course, be set off to any convenient scale.

In the case of a concave lens no real images are obtainable by the method described above; and it is necessary to employ an auxiliary convex lens of sufficiently short focal length to form a converging combination. This should be placed with its axis parallel to that of the bench between the object and the concave lens. The latter being temporarily removed, the convex lens is adjusted until a real image is given, and the position of the screen is noted. The concave lens is placed between the screen and the convex lens, its distance from the screen giving the value of D.

In the manner described above the positions of the focal lines for various obliquities and values of D are now found.

158.

To determine the Focal Lengths of the Components of an Achromatic Lens of Focal Length F, by Observations on Prisms of the Glass used for the Components.—Except in a few anomalous cases, when mixed light is transmitted through a prism, the rays of short wave-length are more greatly deviated than those of longer wave-length. The angle between the directions of the extreme red and violet rays is the "dispersion" of the prism, and when the latter is in the position of minimum deviation, the ratio of the dispersion to the mean deviation is termed its "dispersive power."

If D denote the dispersive power—

$$D = \frac{\mu_v - \mu_r}{\mu - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i.)}$$

The dispersion and dispersive power may be determined on the spectrometer, as explained in Art. **153** (*c*).

Now, if A denote the radius of a circular lens's circumference, f its focal length, and D the dispersive power of its material, the dispersion of the lens is equal to $\frac{A}{f}D$. The distance between the foci for parallel rays of violet and red light is termed the "chromatic aberration."

From the above expression for the dispersion of a lens, it is apparent that, being of equal aperture, a convex and concave lens will together compensate each other's dispersion if their dispersive powers are proportional to their focal lengths.

Thus, in an achromatic combination, if D and f refer to the convex, D′ and f' to the concave lens—

$$\frac{D}{f} = \frac{D'}{f'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii.)}$$

also, if together they are equivalent to a lens of focal length F—

$$\frac{1}{f} - \frac{1}{f'} = \frac{1}{F} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iii.)}$$

Hence, from (ii.) and (iii.)—

$$f = F\frac{D' - D}{D'}, \text{ and } f' = F'\frac{D' - D}{D}$$

In practice the compensation is not made for the extreme limits of the visible spectrum, viz. the red and violet, but for two wave-lengths in the most luminous parts, one in the yellow-orange and the other in the bluish-green. The refractive indices corresponding to these are observed, and used in the numerator of (i.).

159.

To determine the Magnifying Power of a Lens or Combination of Lenses.—The magnifying power of an optical instrument is the ratio of the angle subtended at the eye by the image to that subtended by the object.

The image, and the object also when adjustable in position, is supposed to be at the distance of most distinct vision—usually about 10 inches away from the eye.

In the case of a telescope viewing objects at a distance, so that the rays incident on the object-glass are nearly parallel, the angles subtended are small, and may be considered as proportional to their tangents. The magnifying power is then equal to the ratio of the focal length of the object-glass to that of the eye-piece, and may be calculated from measurements of these lengths.

The more direct method of determination is to fix a scale vertically at as great a distance as convenient from the telescope, provided that its divisions are visible to the naked eye from the latter position, and to focus the telescope upon it. The focus is adjusted until, when viewing the scale directly with one eye and through the telescope with the other, no relative motion occurs between the two images on moving the eyes from side to side. In this condition the scale and its image are at the same distance from the eyes, the latter, of course, inverted.

The number of divisions of the scale, m, which coincide in length with some number, n, of the image is then noted, and hence the magnifying power $\frac{m}{n}$ found.

In the case of a lens or microscope the same method is applied, but a finely divided scale is focussed under the instrument, and another is placed in a parallel position at the distance of distinct vision. The focus is then adjusted until the object directly viewed and the image seen through the instrument are at the same distance from the eyes. The comparison of their graduations is then made.

With a highly magnifying instrument it is often inconvenient to have the scale which is directly viewed as finely divided as the other.

If one of its divisions equals k divisions of the finely divided scale, and m of its divisions equal n of the image, the magnifying power is $\frac{mk}{n}$.

The magnifying power of a lens may be calculated from its focal length. For let an object of length l be placed in the position for distinct vision at a distance d from the eye. The angle subtended is then, taking tangents as measuring angles, equal to $\frac{l}{d}$.

When the object is viewed through the lens of focal length f, and properly focussed, the image is at the distance d from the eye; and if the image be of length L, the angle subtended is $\frac{L}{d}$.

$$\text{Now, } \frac{L}{l} = \frac{\text{distance of image from lens}}{\text{distance of object from lens}} = \frac{d}{u}$$

and if the eye be placed close up to the lens when focussing—

$$\frac{1}{u} - \frac{1}{d} = \frac{1}{f}.$$

$$\frac{1}{u} = \frac{1}{f} + \frac{1}{d}$$

$$\text{therefore } L = ld\left(\frac{1}{f} + \frac{1}{d}\right)$$

$$\text{The angle subtended by } L = l\left(\frac{1}{f} + \frac{1}{d}\right)$$

and the magnifying power is—

$$\frac{l\left(\frac{1}{f} + \frac{1}{d}\right)}{\frac{l}{d}} = \frac{d}{f} + 1$$

This method of calculation is, however, less accurate than that of direct comparison explained above.

160.

To determine the Focal Length and Principal Points of a Lens or Combination of Lenses.—In the case of thin lenses, it is commonly assumed that the deviation of the rays occurs at the centre of their thickness. This assumption, however, is never exactly true, and in the case of thick lenses is inapplicable.

The focal length is consequently not the distance between the focus of parallel rays and the face of the lens or its centre of thickness, but that between the focal point and a certain point on the axis, termed the "principal" or "nodal" point, which may be between, on, or outside the faces, according to the nature of the lens.

As these points are discussed in but few text-books, a short account is here given.

It is generally shown that when parallel rays are refracted directly through a single surface separating a medium of refractive index equal to unity from one having a refractive index μ, the connection between R, the radius of the surface, and F, the focal length, is expressed by the equation—

$$\frac{1}{F} = \frac{\mu - 1}{\mu}\frac{1}{R}, \text{ or } F = \frac{\mu R}{\mu - 1}$$

and that between u and v, the distances of the object and image respectively from the surface, by—

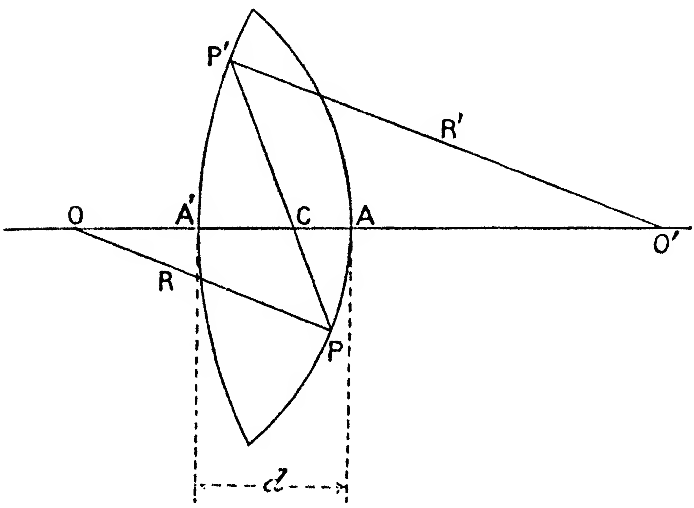
$$\frac{1}{u} - \frac{\mu}{v} = \frac{1 - \mu}{R}$$

also that if L and L′ be the lengths of object and image, then—

$$\frac{L}{u} = \frac{\mu L'}{v}$$

the proper sign being given in each case to the lengths.

Proceeding now to the consideration of a thick lens, let the



thickness measured along the axis from face to face be d, and the radii of the faces themselves be R and R′.

Then the focal length F of the surface of radius R $= \frac{\mu R}{\mu - 1}$, and that F′ of the other $= \frac{\mu R'}{\mu - 1}$.

$$F = \frac{\mu R}{\mu - 1}, \text{ and } F' = \frac{\mu R'}{\mu - 1} \qquad \text{(i.)}$$

The principal, or nodal, points, sometimes termed the optical centres, of a lens are two points on its axis such that a ray of light directed before incidence toward one of them passes after emergence in a parallel direction through the other.

The position of these may be calculated when the radii, thickness, and refractive index of the lens are known. For let OP, OP′ be parallel radii; then OPC, O′P′C are similar triangles, $\frac{OC}{O'C} = \frac{R}{R'}$, and C is a fixed point properly termed the centre of the lens. If S and S′ be conjugate foci, with respect to the two faces, of the point C, they are also by the above definition the principal points. For an incident pencil converging toward S is brought to a focus at C, and emerges from the other face diverging from S′, and having its axis parallel to its original direction.

$$\text{Now, } OO' = R + R' - d$$

$$\text{therefore } OC = \frac{R}{R + R'}(R + R' - d) = R - \frac{Rd}{R + R'}$$

$$AC = \frac{Rd}{R + R'}, \text{ and } A'C = \frac{R'd}{R + R'} \qquad \text{(ii.)}$$

Let now AS, AS′ be the distances, measured inwards, from the faces to the principal points. Then, since C and S, C and S′, are conjugate foci—

$$\frac{\mu}{AC} - \frac{1}{AS} = \frac{\mu - 1}{R}, \text{ and } \frac{\mu}{A'C} - \frac{1}{A'S} = \frac{\mu - 1}{R'}$$

and, substituting from (ii.)—

$$\frac{1}{AS} = \mu\left(\frac{R + R'}{Rd}\right) - \frac{\mu - 1}{R}$$

$$\frac{1}{A'S} = \mu\left(\frac{R + R'}{R'd}\right) - \frac{\mu - 1}{R'}$$

Hence, considering distances measured inwards toward the substance of the lens as positive—

$$AS = \frac{Rd}{\mu(R + R') - (\mu - 1)d}$$

$$A'S' = \frac{R'd}{\mu(R + R') - (\mu - 1)d} \quad . \quad . \quad . \quad \text{(iii.)}$$

The planes perpendicular to the axis through the principal points are the principal planes, and the points in which one is crossed by the direction of the incident, and the other by that of the emergent ray, are equidistant from the axis. For if an object be placed in front of the lens, two images are formed by refraction at the successive faces. Consider the case when object and image are of equal magnitude, and let u and u' be the distances of the object and the first image from the front face, v' and v the distances of the first and final images from the back face.

$$\text{Then } \frac{1}{u} + \frac{\mu}{u'} = \frac{\mu - 1}{R}$$

$$\text{and } \frac{1}{v} + \frac{\mu}{v'} = \frac{\mu - 1}{R'} \quad . \quad . \quad . \quad . \quad . \quad \text{(iv.)}$$

Let L_1, L_1', and L_2 be the linear magnitudes of the object, the first, and final images.

$$\text{Then } \frac{L_1}{u} = \frac{\mu L_1'}{u'}, \quad \frac{\mu L_1'}{v'} = \frac{L}{v}, \quad \frac{L_1}{L_2} = \frac{uv'}{u'v}$$

If the first image be conceived as existing at all points along the axis between the principal planes at S and S'—

$$\frac{u'}{v'} = \frac{AS}{A'S'} = \frac{R}{R'}, \text{ from (iii.)}$$

$$\text{but } \frac{\frac{1}{u} + \frac{\mu}{u'}}{\frac{1}{v} + \frac{\mu}{v'}} = \frac{R}{R'} \text{ from (iv.)}$$

$$\text{Hence } \frac{u}{v} = \frac{u'}{v'}, \text{ and consequently } L_1 = L_2$$

Thus between the principal planes the rays may be regarded as travelling parallel to the axis, and the distance between an object and image of the same size is greater than four times the focal length of the lens by an amount equal to the distance between the principal points. The focal length of the lens is therefore the same, in whichever direction the light passes through it, when measured from the principal point of the back face.

The positions of the principal points, and the focal length as thus defined, may be determined from these relations by a method due to Professor S. P. Thompson.

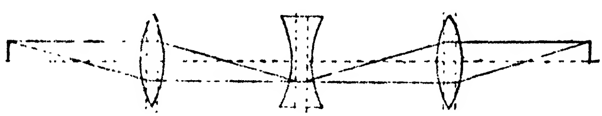
When convex, the lens is mounted on the middle upright of an optical bench, and parallel rays from a collimator are allowed to fall on each side in succession, the positions of the focal points on each side being observed. Similar glass scales are placed at each focal point, and drawn back from them by equal amounts, so that the distance between each scale and its focal point is the same, until the image of one scale coincides exactly in magnitude with the other. The distance between scale and focal point is then on each side equal to the focal length of the lens, and by measuring this distance back from each focal point toward the lens, the positions of the principal points may be marked upon its edge or mounting.

In the case of a concave lens the employment of two auxiliary convex lenses of less focal length is necessary.

Parallel rays from the collimator passing through the concave and a convex lens in succession are brought to a real focus by the latter; and the position of this point being observed, the position of that from which they diverged when incident upon the convex lens may be calculated. This last is the focal point of the concave lens for rays passing through in the one direction, and the position corresponding to it is noted on the optical bench. The concave lens remaining unmoved, the collimator and lens are interchanged, and the position of the other focal point is similarly found and noted on the bench. The point midway between these two focal points is also midway between the two principal points, and should be marked temporarily upon the edge of the lens or its mounting.

Let the two convex lenses be placed before and behind the concave lens at distances from the midway mark slightly less than twice their focal lengths; and at distances exactly twice the focal lengths in front of one and behind the other, let glass scales be placed. Then, the distance between each lens and its scale being unaltered, let each lens and scale be drawn away from the concave lens by equal amounts until the image of one scale coincides exactly in magnitude with the other. The distance apart, D, of the two convex lenses is now observed.

Removing the concave lens, the two convex lenses and scales are brought nearer together until the same conditions as



to the object and image again exist. Let the distance the lenses are apart now be D′.

Then the principal points of the concave lens are at distances $\frac{D - D'}{2}$ on either side of the previously found midway point. These should be marked upon the edge of the lens or its mounting.

Combinations of lenses in fixed relative positions have two equivalent principal points, which may be found exactly as in the case of a simple lens.

161.

To obtain a Pure Spectrum by Means of a Slit, Lens, and Prism.—A narrow slit is cut out in a sheet of card or stiff paper, and the light from a lamp allowed to pass through it. A convex lens and a screen are then adjusted to give a distinct image of the slit. The prism is placed between the lens and screen with its refracting edge parallel to the length of the slit,

and at such a distance from the lens that it intercepts the entire beam of light.

The screen is now moved round the prism until it receives the spectrum, and the position of minimum deviation obtained by turning both prism and screen until the position of the coloured images approaches as nearly as is possible that of the previous uncoloured one.

162.

To map and compare Spectra.—The spectroscope is an instrument essentially identical with the spectrometer described in Art. **153** (*c*), and is put into adjustment as there explained.

In order to produce a greater dispersion, however, a train of prisms is often used instead of but one, and then the position of minimum deviation is secured by placing a single prism on the table and turning it into this position, then adding the second and making the same adjustment for it, and proceeding similarly for each until the train is complete.

In most cases the ends of the prisms are ground square, so that when standing on the table their edge is perpendicular to it, but in others they have levelling-screws attached.

When this is the case, the telescope should be turned to look directly into the collimator tube, and a fine thread stretched across the slit in such a position as to have its image coincident with the horizontal cross-wire.

The levelling-screws are adjusted until the image after refraction through the prism still retains this position. This adjustment is, of course, made for each prism as added.

For all these preliminary operations a sodium flame may conveniently be used. Other substances giving definite lines of known wave-lengths are now placed on platinum foil in the Bunsen burner, and the readings of the verniers noted when the lines coincide with the vertical cross-wire.

The wave-lengths of these lines may be found from tables, and hence a curve plotted with deviations as abscissæ and wave-lengths as ordinates. From this curve the wave-length

of any line may by inferred by observing the corresponding deviation.

Results should be tabulated thus:

Wave-length.	Vernier readings.	Mean vernier reading.	Deviation.

The deviation is the difference between the mean vernier reading and the mean reading when the slit is viewed directly. This last may be noted before the prisms are put in position, or at the close of the observations.

The map of a spectrum is a straight line along which lengths are taken to represent vernier readings, and along which short perpendicular lines are drawn in the positions of the observed spectrum lines.

To obtain the spectrum a spectrum tube may be used; electric sparks may be passed between electrodes of the substance, or its salts may be placed in a Bunsen flame.

The last two methods have the disadvantage that air and other lines are introduced, and when applicable the spectrum tube is by far the most satisfactory. It is connected to the secondary terminals of an induction coil, with its narrow portion parallel to the length of the slit and as near as possible to it.

When electric sparks passing between electrodes of the substance are employed, the electrodes should be vertically opposite and as near as possible to the slit, provided that sparks do not jump across to the tube.

The electrodes are connected to the secondary terminals of an induction coil, and the intensity of the light may be increased by putting the terminals in connection with the inner and outer coats respectively of a Leyden jar.

When it is desirable to compare two spectra, one source of light is placed, as mentioned above, directly behind the slit, and the other at the side, in such a position that its rays are

sent down the collimator tube by the totally reflecting prism covering one half of the slit.

The spectrum then consists of an upper and lower portion contributed by the two substances respectively. Lines continuous vertically through both parts indicate some constituent common to both substances, but when flames or sparks are used, they may, if faint, be air-lines.

163.

To determine the Wave-length of Light from a Given Source by Means of Interference Bands produced by a Biprism.—If a point receive light from two similar sources, emitting waves which are always alike in nature and phase, its intensity of illumination is a maximum when the waves are in phase on arriving there, and is zero when their phase differs by 180°.

The former condition obtains when the difference of the distances between the sources and the point is any whole number of wave-lengths; the latter when it is an odd number of half wave-lengths.

Let the case of two parallel planes be considered, the illumination at any point of the one being due to two parallel luminous slits in the other. If a denote the distance apart of the planes; c the distance between the centres of the two slits; x any distance on the other plane, measured at right angles to the length of the slits, from the perpendicular let fall on it from midway between them; λ the wave-length; and n any whole number; then when—

$$x = \frac{a}{c} n \frac{\lambda}{2}$$

it may be proved that the intensity at x is a maximum or zero, according as n is an even or an odd number.

Consequently, the distance from centre to centre of two bright bands is $\frac{a\lambda}{c}$, and if this be observed, λ may be found.

For, calling the distance w—

$$\lambda = \frac{cw}{a}$$

In practice it is impossible to obtain two real sources satisfying the above conditions, but two virtual sources may be produced by allowing the light from a single narrow slit to fall upon a prism, having its angle nearly 180°. Each half of such a "Fresnel's" prism then deviates the rays in an opposite direction to the other, so that they diverge from two very near parallel lines.

Let d be the distance between the prism and the slit; then very approximately—

$$c = 2d(\mu - 1)\epsilon$$

if ϵ be the refracting angle of the prism, which, of course, $= 90^\circ - \frac{\text{obtuse angle}}{2}$, and μ be the refractive index.

The obtuse angle and μ may be found by means of the spectrometer. Otherwise c may be determined by a method explained below.

To determine λ by this method, one standard of an optical bench is fitted with a slit, behind which the source of light is placed, the centre standard carries the biprism, and the third is fitted with a Ramsden eye-piece and cross-wires so mounted that it can be screwed along horizontally, and its position indicated by a scale and graduated screw-head.

The eye-piece is firstly focussed upon the cross-wires, one of which is turned so as to be vertical. It is next placed so that its axis is in line with the length of the bench, and at the same height above the latter as the middle of the slit, which is also turned so as to be vertical.

The slit and cross-wire may be adjusted to exact parallelism by placing a convex lens of suitable focal length between them (focal length less than one-fourth their distance apart), and illuminating the slit. A real image of the latter may then be obtained in the same plane as the wire, and the wires and slit adjusted to coincidence.

The biprism is now placed in the central standard with its plane face perpendicular to the bench's length, and the eye-piece placed half a metre or less from the slit.

The biprism is adjusted longitudinally, laterally, and vertically, until the bands are seen. It is then turned round a horizontal axis until its edges are parallel with the slit and wire, and the bands attain their greatest distinctness.

The cross-wire is screwed until it is over the centre of a band on one side of the field and the micrometer reading noted. Screwing it toward the other side of the field, the readings at the centres of the second, third, and fourth bands are observed, after which it is moved over to nearly the other side of the field, say to the *n*th band, and the readings at the *n*th, $(n+1)$th, $(n+2)$d, $(n+3)$d, bands noted.

Results should be tabulated thus:

Band.	Reading.	Band.	Reading.	Difference of readings.	w.
1st		*n*th			
2nd		$n+1$			
3rd		$n+2$			
4th		$n+3$			

$$a = \qquad c = \qquad \lambda = \frac{cw}{a} =$$

The difference of readings for bands in the same horizontal line of the table is the width of $n-1$ bands, and the mean value of w as tabulated from these should be taken for the calculation of λ.

The width the virtual images of the slit are apart, denoted by c, may be determined by drawing the eye-piece away from the prism and placing a convex lens between them. The lens will then, in general, give an image of the slits in the plane of the cross-wires when it is in either of two positions. If the distances from centre to centre of slits be measured by the micrometer for the image given in each position of the lens, and these be l_1 and l_2—

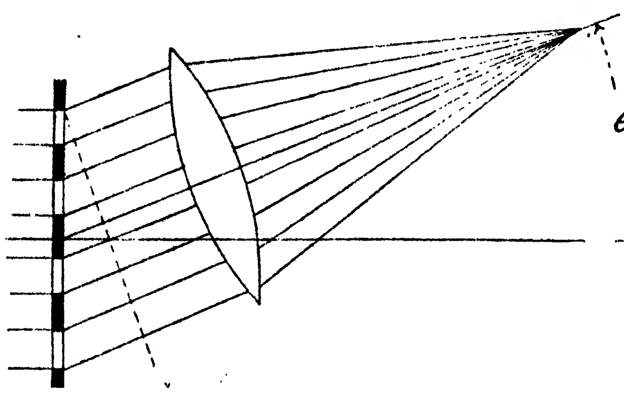
$$\text{Then } c = \sqrt{l_1 l_2}$$

The distance, *a*, between the cross-wires and slit cannot be found directly from the bench readings, as the plane of the cross-wires is not that of the stand vernier. If, however, a rod of known length be placed with one end in contact with the slit and the other in the focal plane of the eye-piece, the difference, *b*, between its length and the bench reading may be noted.

The bench reading, when the bands are focussed, between the eye-piece and slit uprights, diminished by *b*, is then the length *a*.

164.

To determine the Wave-length of Light from a Given Source by Means of a Diffraction Grating.—If parallel rays of light pass through two or more similar apertures, the total resultant intensity in any direction is a maximum or minimum according as the vibrations from similar parts of the



apertures are in the same or opposite phase, *i.e.* according as the difference of path for rays form similar parts of the apertures is an even or odd number of half-wave-lengths. If D be the distance from centre to centre of aperture, and θ the angle between the original direction and that of diffraction,

the difference of path is D sin θ. Hence, if the total illumination be concentrated in a single small region by a lens, the intensity there is a maximum or minimum as D sin $\theta = 2n\frac{\lambda}{2}$, or $(2n + 1)\frac{\lambda}{2}$, and, starting from $\theta = 0$, the positions of the successive bright regions are given by—

$$\sin \theta = 0$$
$$\sin \theta_1 = \frac{\lambda}{D}$$
$$\sin \theta_2 = 2\frac{\lambda}{D}$$
$$\sin \theta_n = n\frac{\lambda}{D}$$

Observing n and θ, D being known, the wave-length λ is therefore given by—

$$\lambda = \frac{D \sin \theta_n}{n}$$

In practice the apertures consist of the spaces between a number of closely ruled parallel lines on a glass plate. The lines are either scratched by a diamond or produced by photography, and may be as many as 40,000 to the inch, though a less number is commonly employed. The lines should be equally spaced throughout the length of the grating, as otherwise the value of θ_n varies according to the part of the grating in use. Accuracy in this respect may be tested by cutting a slit in paper and placing it in front of the grating. As this is passed across the ruled surface, the position of the image in the field of the telescope should not vary.

Reflexion gratings ruled on metal are also employed, but less frequently than those of glass.

The number of lines in a grating is frequently given per Paris inch; the latter equals 2·702 cms. approximately.

The collimator and telescope of a spectrometer are adjusted for parallel rays (Art. 153 (*c*)), and the grating secured

by soft wax on the table. If the table be provided with levelling-screws, the plane of the grating should pass through one of these, and midway between the other two.

The plane is then adjusted till it is perpendicular to the parallel rays, the adjustment about a horizontal axis being made as explained in Art. 162, that about the vertical by turning grating and telescope until a reflected image is seen, the grating being then turned back through half the angle between collimator and telescope. When the table is fixed, or without a vernier, this last adjustment may with sufficient accuracy be made by estimation.

The telescope is now turned to view some image of the slit, and the table-screw through which the plane of the grating passes adjusted until the maximum distinctness is obtained, in which case the lines are parallel to the slit.

The direct reading has been taken before the erection of the grating, and the positions of the first, second, etc., images are now noted on each side of the direct line. Half the angle between corresponding images on the right and left should be taken as the value of θ for that image.

Results should be tabulated thus:

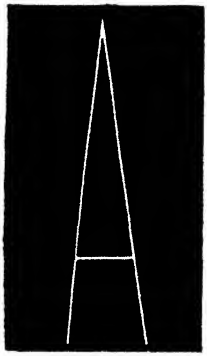
Image.	Right reading.	Left reading.	θ.	λ.

and the mean value of λ finally taken.

In the absence of a spectrometer, and even of an ordinary grating, the following method may be employed:—

A plate of thin glass is smoked over ignited turpentine until an even deposit, sufficiently thick to render it opaque, is obtained, and two nearly parallel but slightly converging lines are ruled with a needle upon the smoked surface. Between these a transverse line, approximately perpendicular to them, is drawn. This plate is placed in a vertical plane before a sodium flame, and the light passing through the clear lines is allowed to fall upon a grating or a piece of fine wire

gauze in a parallel plane, where it is diffracted. The diffracted light is then received in a telescope adjusted for parallel rays. The distance between the grating and the smoked glass is adjusted until the first bands produced by the lines intersect each other across the transverse line. Now, if D denote the distance between the smoked glass and the grating, d the distance from centre to centre of grating spaces, l the length of the transverse line, the θ angle of diffraction, the phase difference of light from successive grating apertures is—



$$d \sin \theta = \frac{dl}{2D}$$

assuming that the sign and tangent of θ are equal, and at the first bright band—

$$\frac{dl}{2D} = \text{one wave-length}$$

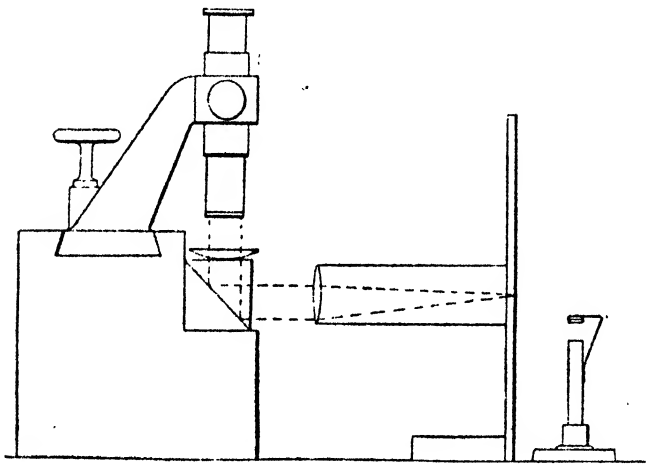
The measurements of l and d are finally made with a reading microscope, or the former may be found by a finely divided scale, and the latter by counting the number of lines in a measured length.

165.

To determine the Wave-length of Monochromatic Light by Means of Newton's Rings.—Newton's rings may be produced by placing a plano-convex lens of considerable focal length upon a plane-glass surface, the lens resting upon the centre of its convex face. If the surfaces are clean, the rings may then be seen, either by reflection or transmission, encircling the point of contact.

For the determination of wave-lengths, the radius R of the curved face is found either optically or by the spherometer, and the lens and plate are placed beneath a reading

microscope. Between the lens and microscope is placed a plate of thin glass at an angle of 45°. Parallel rays from a collimator are thrown horizontally upon this plate, and are thereby reflected down upon the lens, from which they are reflected again vertically through the glass plate into the microscope. The latter being adjusted in position and focus, the diameters of as many dark rings as possible should be



measured by adjusting the cross-wire upon the centre of the ring's width, reading the microscope scale, then similarly adjusting on the other side of the ring and again reading.

The correctness of the readings should be tested, and greater accuracy obtained, by setting out a number of equal lengths as abscissæ, and raising ordinates equal to the squares of the measured diameters of the successive rings. The straight line drawn through the mean positions of the points will then give more exact values of the squares of the diameters than those obtained from the direct measurements. Let D_1^2, D_2^2, D_3^2, etc., denote these squares for the first, second, third, etc., successive rings. Then it may be shown that

$$\text{Wave-length} = \frac{D_1^2}{8R} = \frac{D_2^2}{16R} = \frac{D_3^2}{24R} = \text{etc.}$$

It is advantageous in employing this method to blacken the underside of the plate upon which the lens is placed.

Rings by transmitted light may conveniently be obtained by placing the plano-convex lens horizontally upon one face of a totally reflecting prism, and allowing parallel rays from the collimator to pass horizontally in at the vertical face of the prism, so that they are totally reflected vertically upwards through the lens into the microscope. The diameters of the rings are found and corrected as in the previous case, but the relation between the diameters and the wave-length is now—

$$\text{Wave-length} = \frac{D_1^2}{4R} = \frac{D_2^2}{12R} = \frac{D_3^2}{20R} = \text{etc.}$$

The rings given by transmitted light, though sufficiently distinct for measurement, are fainter than those produced by reflected light.

166.

To compare the Illuminating Powers of Two Sources of Light by Bunsen's Photometer.—This photometer consists essentially of a vertical sheet of paper, one part of which is greased, rendering it semi-transparent. Behind this, in a perpendicular vertical plane, should be a plane mirror, so that each side of the greased paper may be simultaneously visible. This portion of the apparatus is usually fitted to a stand which slides along a graduated photometer bench, the plane of the paper being at right angles to the bench's length.

The two sources of light to be compared are placed on suitable stands at the ends of the bench, one at each end, and vertically on a level with the greased paper.

The position of the latter is then adjusted until the greased portion, as seen in the mirror, is neither brighter nor darker than the other part of the paper, and its distances from the sources is noted.

Let I_1 and I_2 denote the intensity of the sources and, d_1 and d_2 be the distances of the greased paper from them. Then—

$$\frac{I_1}{I_2} = \left(\frac{d_1}{d_2}\right)^2$$

This expression refers to the intensity in the direction of the paper. Where it varies in different directions, the source should be rotated about a vertical axis, and comparisons made for various positions. It is generally impracticable to make determinations for the various directions in a vertical plane.

To express the intensity in candle power, one of the sources must be a standard sperm candle burning 120 grains per hour.

This method is only applicable in cases where both sources emit light of approximately the same colour.

As exposure to strong light incapacitates the eye for estimating equalities of intensity, all such should be avoided before and during the comparisons. Movement of the grease-spot to and fro on either side of the balancing position enables the latter to be determined with greater accuracy.

167.

To determine the Reflective Power of a Surface.—The reflective power of a surface is the ratio of the intensity of the reflected to that of the incident light, when the rays are parallel and unpolarized.

In general it varies with wave-length and with the angles of incidence and reflection.

It is convenient to enclose the source of reflected light in a metal box furnished with a window, so that, except when on the optical bench and turned toward the greased screen, its rays shall not fall on the latter. This is balanced (Art. 166) at some distance D from the screen against a standard source of light, and then removed from the bench. The reflecting surface is now placed somewhat near to the screen, and the enclosed

light on a line making some angle θ with the axis of the bench. The light and the reflecting surface are adjusted until the reflected rays fall upon the greased screen as before, and the distance between the two former is varied until balance is obtained again with the standard.

Let the sum of the distances between light and surface, and surface and greased screen, be D. Then the reflecting power for light incident at an angle $\frac{\theta}{2}$, θ being measured toward the bench's centre, is —

$$\frac{d^2}{D^2}$$

The distance between the standard light and greased screen must, of course, remain unchanged.

The determination should be made for as many angles as possible between 0° and 90°.

Results should be tabulated thus:

D.	θ.	d.	Reflective power.

168.

To compare the Intensities of Light by Rumford's Photometer.—This photometer consists of a vertical rod placed in front of a screen. The two sources of light to be compared are placed in such positions that the shadows of the rod cast on the screen by each come together, and one source is then brought nearer to, or removed from, the screen until each shadow is of equal darkness. This condition being attained, the intensities of the lights are proportional to the squares of their distances from the screen.

The screen should be adjusted perpendicular to a straight

line passing through the rod and bisecting the angle contained by lines joining the rod to the lights, and comparisons should be made with various distances between the sources and screen.

169.

To determine the Coefficient of Absorption of a Substance for Light.—The coefficient of absorption is the fraction of the total light absorbed, or the logarithm of the ratio of the entrant to the emergent intensity, when light is transmitted through a plate of unit thickness. Denoting it by k, and letting I, I′, I″ indicate the incident, reflected, and transmitted intensities—

$$k = \log_e\left(\frac{I''}{I - I'}\right)$$

or, since, if c denote the reflective power of the substance's surface, $I' = Ic$—

$$k = \log_e\left\{\frac{I''}{I(1 - c)}\right\}$$

The value of the coefficient varies with the wave-length.

The source of transmitted light is allowed to radiate directly to the greased screen, and a balance obtained, at some distance D from the latter, with a standard light. The determination of the reflecting power should be made at the smallest possible angle of incidence by the method of the last article, and finally the plate of substance is placed perpendicularly to the bench's axis between the source and screen. The distance D′, between source of light and screen, at which the transmitted balances the standard is then found. Since—

$$\frac{I''}{I} = \left(\frac{D'}{D}\right)^2$$

$$k = \log_e \frac{D'^2}{D^2(1 - c)}$$

If two plates of different thicknesses, l_1 and l_2, but similar otherwise, be available, the error introduced by diffusion, and the determination of c, may be avoided. For if radiating directly to the screen, the source of light balances the standard when the distance between the screen and itself is D, with the thinner plate of thickness l_1 interposed at distance D′, and with the other plate of thickness l_2 substituted at a distance D″—

$$\text{Then } k = \frac{\log\left\{\left(\frac{D}{D''}\right)^2 - \left(\frac{D}{D'}\right)^2\right\}}{l_2 - l_1} = \frac{2(\log_e D' - \log_e D'')}{l_2 - l_1}$$

The coefficient of transmission is the ratio of the intensity of the transmitted to that of the incident light for a plate of unit thickness. Its relation to the coefficient of absorption is expressed by the equation—

$$\text{Coefficient of transmission} = \epsilon^{-k}$$

170.

To determine the Refractive Index of a Substance by Observation of the Polarizing Angle.—Ordinary light after reflection is more or less polarized in the plane of reflection. The angle of incidence at which this effect is a maximum is termed the polarizing angle, and is such that its tangent is numerically equal to the refractive index of the reflecting substance.

The substance should be placed on the table of a spectrometer with one of its plane faces vertical and passing across the centre of the table.

An analyzing Nicol is fitted in the telescope, and turned until its principal plane is parallel to that of reflection.

The table and telescope are turned until the reflected light is seen in the field. As the table is turned still further round, and followed by the telescope, the angles of incidence and reflection diminish, the intensity of illumination falls off, and a

position of maximum extinction is reached, beyond which the illumination again increases.

The angular distance between the collimator tube and the telescope when the maximum extinction occurs is evidently twice the polarizing angle.

The general adjustments of collimator, telescope, etc., are as described in previous articles. In the absence of a mark indicating the principal plane of the analyzer, it may be turned to its correct position by placing the table and telescope at such an angle that the illumination varies as the analyzer is rotated round the axis of the telescope, and turning it to the position of least illumination.

After making an observation with the light thrown to one side of the spectrometer, the table should be turned so as to throw it on the other side and the observation repeated, the mean of the two being afterwards taken.

In the case of liquids, a microscope furnished with an analyzing Nicol is frequently employed, the liquid being placed on the stage, and the angle made by the tube with the vertical, *i.e.* the angle of reflection, indicated by a graduated sector. This apparatus is equivalent to a spectrometer in a vertical plane, and the method of procedure is as explained above.

The angle of polarization and the refractive index of a substance vary with the wave-length, so that it is necessary to employ a monochromatic source of light.

171.

To determine the Molecular Rotatory Power of a Substance in Solution.—When plane polarized light is transmitted through solutions of certain organic substances, its plane of polarization is rotated through an angle, the magnitude of which is dependent upon—

(*a*) The nature of the substance.

(*b*) The concentration.

(*c*) The length of the path traversed in the solution.

(*d*) The wave-length of the light.

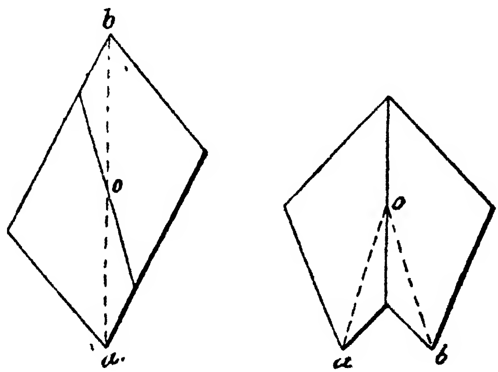
(*e*) The temperature.

In some substances the rotation viewed in the direction in which the light travels is right-handed or positive; in others, it is left-handed or negative.

The "molecular rotatory power" is the angle of rotation produced when the light passes through unit thickness of solution containing the active substance in unit density, *i.e.* 1 gram of the substance per cubic centimetre of solution.

Instruments for determining the amount of rotation, termed saccharimeters, are of various forms, the following being some of the more important:—

Jellett's.—The light is in this polarized by reflection or by



transmission through a Nicol. After passing through the solution, it falls upon the analyzer, the special construction of which is the peculiar feature of the instrument.

A rhomb of Iceland spar, about 2 inches long, has its ends cut off square, and is divided into two parts by a plane at right angles to its ends, and making a small angle with their longer diagonal. One of these parts is inverted, and both are then cemented together.

The ordinary ray then passes undeviated through the halves, polarized in planes perpendicular to *oa* and *ob*, while the

extraordinary rays are deviated to the sides, and in prisms of sufficient length do not emerge from the end face. Hence, when plane polarized light is incident, the component of vibration parallel to *oa* is transmitted by one half, and that parallel to *ob* by the other, so that the intensity of illumination of each half is the same only when the incident light is polarized in the plane of division. The rotating part, which may be either the polarizer or analyzer, is therefore adjusted until this condition exists. The light employed with this instrument need not be monochromatic.

Laurent's.—The polarized light before passing through the solution falls upon a plate one half of which is quartz or selenite cut parallel to the axis, of such thickness as to introduce a quarter wave-length phase difference between the ordinary and extraordinary rays. The other half of the plate is of glass. The light emerging from the halves of the plate is polarized in different planes, and when transmitted through a Nicol, illuminates the two halves of the field equally only when the principal plane of the Nicol bisects the angle between the planes of polarization.

Owing to the quarter wave retardation this instrument can only be used with monochromatic light.

The Biquartz.—This simple piece of apparatus consists of a circular plate of quartz made up of two semicircular pieces cut at right angles to their axes. One of these rotates the light right-handedly, the other left-handedly, so that on emergence the light from them is polarized in different planes. When viewed through a Nicol, the halves of the field have the same tint only when the principal plane of the latter bisects the angle between the planes of polarization. When suitably cut, the biquartz then yields a neutral tint termed the "tint of passage," a small rotation in either direction turning one half red and the other blue, when white light is used.

Wild's.—The light, which must be monochromatic, is in this form caused to converge, after polarization, through a Savart's polariscope. This consists of two thin plates of uniaxial crystal superposed, with the principal sections at right angles. When viewed through an analyzer, the field is in

general crossed by interference bands, which vanish in two positions of the analyzer 180° apart. Owing to the convergence, an eye-piece is necessary, and this is focussed so that the bands are clear and distinct.

Poynting's.—Polarized light is transmitted through a cell of some rotating solution. Across one half of this cell is placed a glass plate, so that the light in one half of the field traverses a greater thickness of liquid than that in the other. Hence, as in several of the previous forms, the planes of polarization in the halves of the field differ, and the halves only appear of the same intensity when the principal plane of the analyzer bisects the angle between those planes.

Of course the liquid in the cell is quite distinct from that of which the rotation is to be measured, the latter being placed between the cell and the analyzing Nicol. Another less simple form of saccharimeter has also been proposed by Poynting.

In all these instruments the rotation is measured by means of a graduated disc attached either to the polarizing or analyzing piece.

As a source of light the straight filament of an electric lamp is satisfactory, and its rays may be monochromatized by transmission through a plate of potassium chromate. For monochromatic work, however, a sodium Bunsen flame is very commonly employed.

The solution to be observed is contained in tubes furnished with screwed plane-glass ends, which are placed between the polarizer and analyzer.

In making a determination, the tube is filled with pure solvent, the eye-piece focussed, and the graduated disc turned until the appearance proper to the particular instrument is obtained. Its position is then noted. Turning the disc through 180°, it is again adjusted, and the reading observed.

Let the mean of the first reading and the second (diminished by 180°) be θ_1.

In many cases the required appearance exists over a considerable angle, and in such the mean position between the limits should be taken as the correct one.

A weight W of the active substance is dissolved, forming V

cubic centimetres of solution, with part of which the tube is filled, and the previous observations are repeated.

Let the reading now be θ_n.

Proceeding in this way with increasing values of $\frac{W}{V}$, the rotations are measured, and finally the length L inside the tube.

Results should be tabulated thus:

$\frac{W}{V}$	θ_1.	θ_n, etc.	L.	$\frac{\theta_n - \theta_1}{L}$

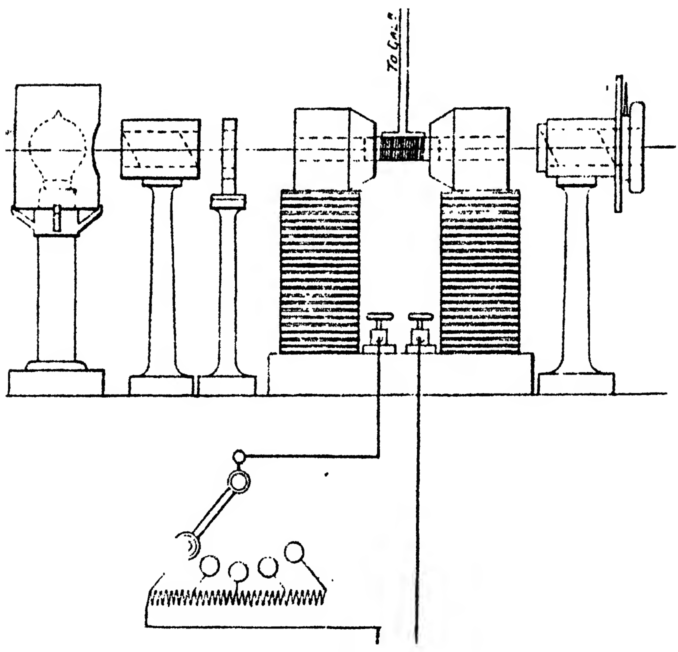
A curve should be plotted with values of $\frac{W}{V}$, *i.e.* the density of the active substance, as abscissæ, and of $\frac{\theta_n - \theta_1}{L}$ as ordinates. From this the value of the ordinate when $W = V$ may be found.

172.

To determine the Relation between the Rotation of Plane Polarized Light passing through a Substance in a Magnetic Field and the Field Strength.—The magnetic field may be produced by an inverted horse-shoe electro-magnet connected to a battery circuit containing an adjustable resistance whereby the current may be varied. To each pole of the magnet a pole-piece is attached, and bored with a hole such that a beam of light, suitably directed, can pass through each in succession; while, to obtain concentration of the field in the interpolar gap, they should also be coned down toward each other. Into the hole in the pole-pieces is inserted the substance, the rotation in which is to be observed, in the form of a short bar with plane ends. This is pushed through until it bridges across the gap, resting in the pole-pieces at the ends;

and insulated wire is wound closely round it as many times as convenient, the ends being then connected to a ballistic galvanometer. A source of light, a polarizing Nicol, a biquartz, the pole-piece hole, and an analyzing Nicol furnished with a graduated circle for measuring rotation, are then arranged in line, so that the light passes through each in succession.

The analyzing Nicol is turned until each half of the field is



of the same colour, the tint of passage, and its reading noted. It is then turned through 180°, and the same adjustment made.

A small current is now sent round the magnet, and the throw given by the galvanometer observed, after which the analyzing Nicol is again adjusted, and the two readings noted.

The magnetizing current is increased by successive sudden steps, and the same observations made for each, until the maximum excitation is reached.

Results should be tabulated thus:

Galvanometer throw.	Analyzer readings.		Field strength.	Rotation.

The field strength values are only relative, and are obtained by adding all the preceding entries of the galvanometer throw. The rotation is the difference between the mean of the two analyzer readings and its original reading when the magnets were unexcited.

A curve should be set out with values of the field strength as abscissæ and those of the rotation as ordinates.

PART V

SOUND AND OTHER VIBRATIONS

173.

To determine the Frequency Difference between Two Forks of nearly Equal Pitch by counting the Beats.—When two sources of sound simultaneously produce waves differing but slightly in length, interference occurs, and an alternation of loud and feeble sound is heard.

One such increase and subsequent waning is termed a "beat." If N wave-lengths from one source have the same total length as $N + 1$ from the other, it is obvious that the interference cycle is completed during the time of N vibrations of the first, and that one beat occurs in the same time.

Hence, if mN be the frequency, *i.e.* vibrations per second, of the lower fork, that of the other is $m(N + 1)$, and there are m beats per second. The number of beats per second, therefore, equals the difference of the frequencies.

Provided these beats are not too few, or too many, per second, they may be counted when both forks are sounded together. If the forks are very nearly indeed of the same frequency, the vibrations die out before a sufficient number have been produced; if their difference be too great, the beats are produced more rapidly than they can be counted. Beats of considerable frequency, however, may be counted by starting a stop-watch at a beat, counting them in fours or sixes, making a mark on paper at every fourth or sixth, and stopping the watch at a beat corresponding to a mark. The number of marks multiplied by the number of beats in a group is the

whole number in the recorded time, if that beat at which the watch is started be counted as o.

It can generally be decided by ear which fork has the greater frequency. Otherwise a small piece of sheet metal or of wax may be attached to a prong of one fork, and the beats again counted. By reducing the load and trying it with each fork, the number of beats may be diminished. When this is the case, the load is upon the fork of higher pitch.

If the difference of frequencies be too great for beats to be heard directly between the forks, a third fork of intermediate frequency may be employed. The frequency difference between the two is the sum of the differences between each and the accessory fork.

The application of this device is also the best method of adjusting two forks to exact unison. By adding a small load to that one of higher pitch, the rate of beats given by each with the third is made equal, and they are then of the same frequency.

By filing away metal from the end of the prongs, and so reducing their moment of inertia about the base, the pitch may be permanently raised; but this should never be attempted by the student, without express instructions.

174.

To determine the Velocity of Propagation of Sound in a Gas by Means of a Resonating Column and a Fork of known Frequency.—The sound of a vibrating fork held at the open end of a closed tube is greatly increased when the length of the tube, measured from the point of reflection at the open end, is an odd number of quarter wave-lengths of the waves created by the fork.

In the case of circular tubes the point of reflection is situated about o·4 of the diameter outside the end; and, in general, its distance beyond it approximately equals the sectional area of the tube divided by four times the radius of the opening.

If, beginning with a very short length of air-column, it be gradually increased, the intensity of the sound successively rises to a maximum, falls to a minimum, and rises to a second maximum.

The difference between the lengths producing maximum resonance is half the length of the wave produced by the fork, and is theoretically the better measurement of the same. Owing, however, to the second maximum being far weaker than the first, it is in practice preferable to find the length corresponding to the latter, add the correction for the open end, and so obtain the quarter wave-length.

The wave-length and frequency being known, the velocity is equal to their product.

The simplest form of resonating tube is a tall jar, the air-column within which may be adjusted by pouring water in or out. This is applicable in the case of air and gases denser than air, but for determining the velocity in lighter gases, a tube with a movable plug and its open end at the bottom is required.

As there is always considerable difficulty in selecting the exact length at which the resonance is a maximum, a considerable number of observations should be made and the mean value adopted.

If the increase of velocity due to a rise in temperature is to be found, the tube must be jacketed with water or steam, and the observations repeated for the various temperatures.

The correction for the open end of a given tube may be experimentally found if the first two lengths, L_1 and L_2, of maximum resonance are determined; for evidently—

$$\text{End correction} = \frac{L_2 - L_1}{2} - L_1 = \tfrac{1}{2}(L_2 - 3L_1)$$

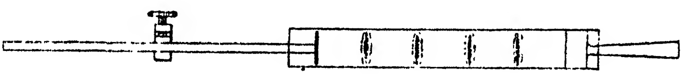
175.

To compare the Velocities of Sound in Air and in the Substance of a Rod by Means of Kundt's Tube.—The usual

form of Kundt's appliance is a glass tube of about 5 cms. diameter and of about a metre's length. One end is closed by a cork, which tightly grips the centre of the rod, and the other is closed by a cork furnished with a handle, whereby its position in the tube may be adjusted. To the inner end of the rod is fixed a disc, which just clears the inside of the tube. A preferable arrangement is to clamp the centre of the rod independently to a bench, the disc being a short distance inside the tube, and the end of the latter being open.

Cork dust or lycopodium is placed in the tube, and distributed as evenly as possible in a thin layer between the disc and cork plug.

The tube being now clamped or supported conveniently in



such a way that the disc on the rod can vibrate freely, a cloth is drawn along the outer half of the rod, producing longitudinal vibrations, and causing it to squeak.

The cloth should, in the case of wooden or metal rods, be sprinkled with resin, or, in the case of glass, with alcohol.

The vibrations of the air inside the tube cause the dust to redistribute itself in ridges, and the movable plug should be adjusted in position until narrow groups of transverse ridges are well defined at intervals. Rotation of the tube while the cloth is drawn along the rod facilitates this redistribution, but is not usually necessary.

Now, the wave-length in the rod is twice its length, and that in the air is twice the length between two groups of ridges, the frequency being the same for both. Hence—

$$\frac{\text{velocity in material of rod}}{\text{velocity in air at T}^{\circ}} = \frac{\text{length of rod}}{\text{length between ridges}}$$

As even when the natural periods of the rod and the air-column are considerably different forced vibrations are

produced, the mean of several independent observations should be obtained.

The distance from centre to centre of the groups of ridges should be found by measuring the length of tube between one and the nth succeeding one, then dividing by n.

Since the velocity of sound in a substance satisfies the equation—

$$(\text{velocity})^2 = \frac{\text{Young's modulus}}{\text{density}}$$

this modulus may be determined if the velocity in air be known, and the density found by measuring and weighing the rod.

176.

To determine by Means of a Sensitive Flame the Pitch of a High Note.—A sensitive flame is produced by igniting gas passing through a pinhole burner under a pressure of not less than 8 inches of water, preferably 9 or 10.

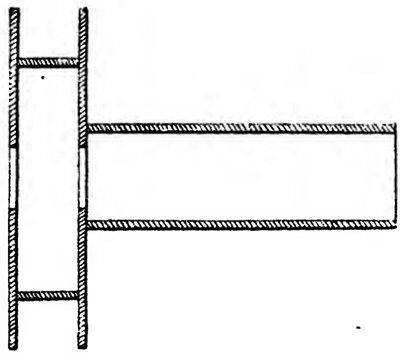
Such a flame flares when fully turned on; but the supply of gas may be so adjusted that it burns quietly except when distributed by sound-waves. It is necessary, however, that the sound-waves should be of short lengths, such as those emitted by a shrill whistle or by hissing.

The pressure required for such a burner necessitates the employment of a weighted gas-bag; but in cases where this is inconvenient, the flame may be obtained by placing the burner below a piece of fine gauze, and using the gas at the pressure of the supply. The gauze should be sufficiently large to prevent the gas below it burning when that above is ignited, and the distance between the burner and gauze is adjusted until the flame is steady in still air, but flaring when disturbed by the sound.

It is advantageous to surround the flame by a glass or metal tube about 2 inches in diameter and 9 inches in length.

The most suitable sources of sound for this determination

are a shrill whistle, a squeaking reed as fitted in many toys, or a bird-call as shown. If such a source be placed opposite a large vertical plane surface, such as a wall, at a distance of several feet, the directly radiated waves and those reflected from the surface interfere, producing stationary vibration along the line passing perpendicularly to the surface through the source. Nodes, therefore, occur along this line at intervals of half a wave-length, the first being at this distance from the surface. The position of the source itself is in general neither at a node or antinode. Owing, however, to the reflected waves being of slightly less amplitude than those directly radiated, these nodes are not places in which the air is absolutely devoid of motion, but are the positions in which minimum movement occurs.



When the sensitive flame is moved along the line, these nodes are indicated by its burning quietly, flaring being produced at all intermediate positions.

The total length of a number of segments being then measured, that of one, *i.e.* a half wave-length, may be found with tolerable accuracy.

The form of flame firstly described should be so adjusted that the top of the burner is situated on the line; with the second form, the line should pass between the burner and the gauze.

177.

To determine the Effect on Transversely Vibrating Strings of Variations in Length, Mass, and Tension, by Means of a Sonometer.—A sonometer consists of an elongated box on

which are two strings or wires. One of these is fixed at one end, and strained tight by a screw at the other. The other string is fixed at one end, and strained by being passed over a pulley and loaded at the other. Each has its vibrating length adjustable by means of two bridges.

(*a*) *To determine the Relation between Pitch and Length.*—The two strings are made the same length by moving the bridges, and brought to the same pitch by adjusting the tensions of the screw and weights.

The bridge under the weighted string is then moved until it emits a note an octave higher than the other, *i.e.* until its frequency is doubled.

The lengths being noted, the ratio of the frequencies should be compared with the inverse ratio of the lengths.

The wire should be shortened to give the double octave, and the same comparison again made.

(*b*) *To determine the Relation between Pitch and Tension.*—The two strings, being the same length, have their tensions adjusted so that each gives the same note, which should be as low as convenient. The load on the weighted one is then increased until its note is an octave higher. The square of the frequency ratio should be compared with the ratios of the weights. This should be repeated for various values of the original load.

(*c*) *To determine the Relation between Mass and Pitch when the Total Stretching Force is Constant.*—A string, of mass m per unit length, is adjusted to unison with the screwed-up string, and its length, l, noted. Another string of the same material, but of mass m', is substituted, and loaded with the same weight. The length, l', of this when brought to unison with the screwed-up string is noted.

The pitch of the two wires if their masses per unit length were equal would be given by—

$$\frac{\text{pitch of first}}{\text{pitch of second}} = \frac{l}{l'}$$

The square of this pitch ratio should be compared with the ratio of the masses per unit length.

The mass per unit length, for which the weight may be substituted, is found by weighing a known length of each string.

(*d*) *To verify the Formula of a Vibrating String by Comparison with a Fork.*—The relation between the frequency, length, stretching force (in dynes), and mass per unit length is expressed by the equation

$$\text{frequency} = \frac{1}{2L}\left(\frac{\text{force}}{\text{mass}}\right)^{\frac{1}{2}}$$

where L is the length.

To verify this, it is only necessary that the string should be turned to unison with the fork, and the particulars observed.

A slight discrepancy is to be expected owing to the string not being perfectly flexible, the effect of which is to slightly increase the frequency.

178.

To determine the Frequency of a Note by Comparison with that of a Syren.—A syren essentially consists of a plate perforated with holes at equal distances apart and moving in front of an air-jet.

The usual form is a cylindrical brass box, the top of which has a circle of equidistant holes drilled through in a sloping direction. In contact with the top is a circular plate fixed to a vertical spindle, and furnished with a circle of holes similar in size and number to those in the box, but inclined in the opposite direction.

The spindle is connected above to gearing which registers the number of revolutions made in any interval during which a knob is depressed. At the bottom of the box is a pipe into which air is blown by bellows.

Owing to the opposite inclinations of the holes in the rotating and fixed plates, the former, when started, is maintained in motion by the puffs of air escaping and its velocity is regulated by increasing or diminishing the pressure from

the bellows. The sound is produced by the periodic escape of air from the holes, its pitch being equal to the product of the number of holes in the ring and the revolutions per second.

The method of driving by air is unsatisfactory, though common, for it necessitates the intensity of a note increasing with its pitch, and two sounds can only be compared with each other when of about the same loudness. It is preferable to drive the syren independently of the air-blast by means of a motor, the speed of which may be finely adjusted by a weighted band on the spindle.

In making a determination, the note is sounded and the speed of the syren increased until its fundamental note is of nearly the same pitch, and beats occur. Considerable care is necessary not to mistake beats given by the upper partials of the syren for those of the fundamental note, the latter not being always the loudest.

The true beats between the note and the syren's fundamental being heard, and the velocity being steady, the counting gear is thrown into action for a noted interval of time, and the beats during the same counted. The revolutions of the syren are given by the difference of the dial readings before and after the interval.

If these be N, the length of interval T seconds, and m the number of holes in the syren—

$$\text{Frequency of syren} = m\frac{N}{T}$$

and if n beats occurred during the time T—

$$\text{Frequency of given note} = \frac{1}{T}(mN + n)$$

The most convenient note to compare with that of the syren is one given by a pipe, and below the middle C. The note of a fork is of such different quality that the comparison is very difficult; it should, however, be attempted after experience with a pipe.

179.

To determine the Frequency of a Fork by Means of the Dropping Plate.—The plate, which is commonly of glass, is covered with a uniform layer of soot on one side by being held over burning turpentine or camphor, and is suspended by a silk thread from the top of the upright which guides its descent. The position should be such that the style on the prong of the fork, which is securely clamped in front, is just in contact with the plate above its lower edge, the fork itself being set to vibrate in a horizontal plane.

These adjustments being made, the fork is strongly bowed and the supporting thread ignited, thereby liberating the plate, which in its fall receives an undulatory marking from the style. For setting the fork in vibration, instead of a bow a small metal plate having a rectangular notch cut out of one side is

often used; this notch is sufficiently wide to clip the prongs of the fork when they are sprung inwards, and when pulled off suddenly leaves them in vibration.

From the extreme lower end of the line traced, two lengths are measured off, such that their square roots are in the ratio of two to one. These may conveniently be 2 centimetres, and 8 centimetres. Along the last six centimetres, between the two marked positions, the number of complete waves can usually be counted without difficulty. Let this number be N.

Now, it follows from the laws of falling bodies that the first two and the last six centimetres were fallen through, in equal intervals of time; therefore the fork made N vibrations while the plate fell two centimetres, that is, in 0·064 seconds.

The frequency of the fork, with the style attached, is then equal to $\frac{N}{0{\cdot}064}$.

180.

To determine the Frequency of a Fork by Means of an Electric Chronograph.—The chronograph consists of a cylinder which is rotated by a handle or by a cord from a motor, and which usually has a screw cut on its spindle causing it to travel axially.

This is wrapped round with a sheet of paper smoked over burning camphor, and fixed on the cylinder by springs, rubber bands, or gum. The fork is firmly clamped in a standard, and so adjusted that a style on one of its limbs is just in contact with the smoked paper, its motion being parallel to the axis of the cylinder.

The cylinder, or drum, and the fork are insulated electrically from each other, and each connected to a secondary terminal of an induction coil, the primary circuit of which is so arranged that it is closed momentarily when a pendulum passes its central position, or when a balance wheel is at the limit of its swing. This is accomplished by connecting one primary terminal of the coil to the pendulum or wheel, the other to one pole of the battery, and the other pole of the battery to a drop of mercury suitably placed in relation to the oscillating part. A spark then passes between the style and drum, knocking a spot of black off the paper, every time the primary is closed. Any small error due to the mercury bead not being exactly in its correct position may be avoided by ignoring the alternate spots, and considering the distance from any one to the next but one as the movement of the drum during an interval equal to the periodic time of the pendulum. In the case of a balance wheel, where contact is made at one end of the swing, this is unnecessary, the interval between two successive spots being equal to the period.

The smoked paper being carefully fitted round the drum and all connections made, the pendulum is set in motion, the

fork made to vibrate, and the drum rapidly rotated. A convenient means of vibrating the fork is the slotted piece described in Art. 179.

The style traces an undulating line on the smoked paper, each complete undulation corresponding to one vibration of the fork. On this line are also the spots corresponding to the pendulum's vibration, so that the periods of fork and pendulum may be at once compared by counting the waves between the spots.

The period of the sparking being observed, or calculated from the known dimensions of the pendulum, that of the fork is obtained.

181.

To compare the Frequencies of Two Forks having nearly the Same Pitch by Means of a Vibration Microscope.—The vibration microscope consists of a third fork, the pitch of which is adjustable by means of masses sliding on its limbs. To this is attached a small convex lens, vertically above which is an eye-piece. The forks to be compared have a small bright point scratched on the end of a prong, or the end may be smoked and a small white spot marked upon it. One of these is placed in a clamp under the lens in such a way that the vibrations are in a vertical plane, and the spot under the centre of the lens. The eye-piece is focussed until a distinct image of the spot is seen, and when possible a condensing lens should illuminate the latter.

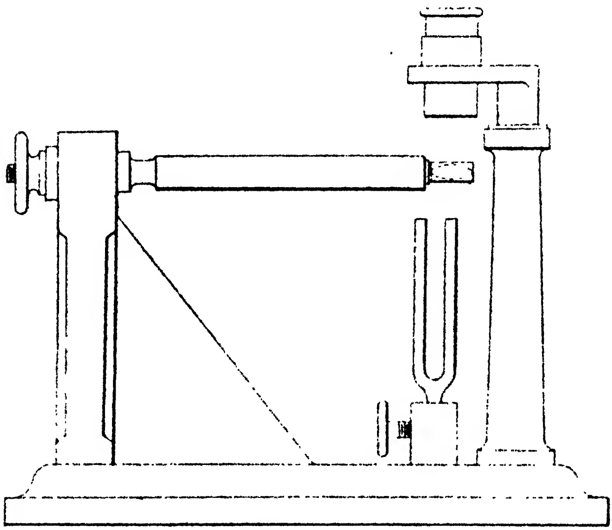
The loads on the auxiliary fork, which vibrates horizontally, are then adjusted in position until its pitch is slightly lower than that of the forks to be compared.

Both are then set in vibration, producing Lissajous' figures, which are observed through the eye-piece. The curves change their form, becoming straight lines at intervals.

The interval between the occurrence of two successive straight lines inclined in the same direction is that in which the vertical fork gains one complete vibration, and they consequently have the same frequency as the beats, but are susceptible

of more accurate observation. Hence, by noting the number of times this occurs in a certain period, the frequency difference of the forks is found.

The other of the two forks to be compared is then substi-



tuted, and the difference in frequency between this and the lens fork similarly observed.

The smaller difference is deducted from the greater, the remainder being that between the forks themselves.

182.

To determine the Frequency of a Fork by Means of an Auxiliary Fork and a Pendulum.—The most accurate method of determining the absolute frequency of a fork's uncontrolled vibrations is to compare it with another under the vibration microscope, as explained in Art. 181. The frequency of this auxiliary fork, which may conveniently be the lens fork itself,

should be approximately some simple measure of that which is to be standardized, and its own vibrations are compared with those of a pendulum of which the period can be very accurately determined.

The necessity for the second fork, which will here be termed the interruptor fork, arises from the fact that, for comparison with the pendulum, a small plate of thin sheet metal must be attached to the end of each prong. These plates are so fixed that in the position of equilibrium they overlap without contact, drawing apart, however, when the fork vibrates so as to leave a narrow slit between them once in each vibration (see Fig. of Art. 183). The comparison with the original fork is, of course, made after their attachment.

The pendulum, which, for convenience, should be of some simple period, not greater than ten times that of the interruptor fork, has a silvered bead near its free end. Its oscillations should be maintained by clockwork or otherwise, to prevent diminution of amplitude, which would introduce error, and a strong light should be directed upon the path of the bead.

The interruptor fork is placed at a convenient distance in front of the pendulum at about the same level as the bead, and with its plane of vibration parallel to that of the latter.

Upon setting both fork and pendulum in motion, and placing the eye close to the plates, a number of distinct and separate images of the bead are seen owing to persistence of vision. If the frequency of the pendulum be p, and the fork make n vibrations while the pendulum makes one, the number of images is in general n. If n be an exact whole number, they are stationary; but if, as is usually the case, it be not, the images fall into two groups, which move across the field in opposite directions, one group being the positions of the bead in the forward, the other in the backward swing. The opposite movement is the result of each being seen a little earlier or later in successive oscillations.

A preliminary observation should be made, placing a screen between the fork and pendulum intermittently, at each swing in the same direction if necessary, to ascertain whether the group seen moves in or against the direction of the bead's swing. In

the former case the frequency of the fork is less, in the latter greater, than n times that of the pendulum.

A convex lens should then be placed between the resting position of the bead and the fork, so as to focus a small image on the slit between the plates. In front of these a microscope is adjusted to receive the rays when transmitted.

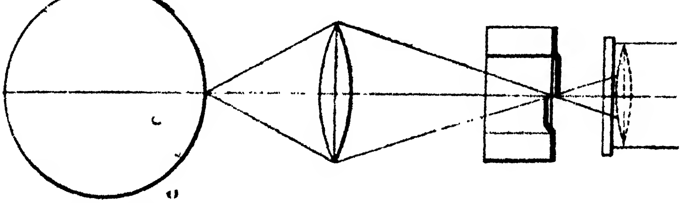
Both fork and pendulum now being set in motion, the images are seen to cross the field at intervals in opposite directions, and by means of a stop-watch the interval during which a number, m, cross in the same direction is noted. Let this interval be T. The fork has then made $np\mathrm{T} \pm m$ vibrations in this time, and its frequency is given by—

$$\text{Frequency of fork} = np \pm \frac{m}{\mathrm{T}}$$

the positive or negative sign being taken according as the preliminary observation showed the frequency to be greater or less than np.

183.

To determine the Frequency of a Fork by Means of a Stroboscope.—The stroboscope is either a disc or cylinder bearing on its face or circumference respectively rings of alternate black and white intervals. Several such rings are usually carried, the number of intervals being different in each. When the



stroboscope is in rotation, these rings, viewed directly, appear of a uniform grey tint. If, however, they be looked at through the plates of a fork such as that described in Art. 182, a distinct series

of dark and light intervals is seen. If the actual spaces on the stroboscope are parallel bands, they appear as triangular teeth when seen through the fork. Let the frequency of the fork be n per second, the revolutions of the stroboscope be p in the same time, and the number of dark bands in the ring under view be q. Then if n be exactly equal to pq, the position of the teeth is stationary; but if n be greater or less than pq, they appear to move against or in the direction of the stroboscope's rotation respectively. The time during which a tooth moves through its own width is that in which the fork gains or loses one vibration relatively to a body moving with a frequency pq.

To determine this period a convex lens should be placed between the marked ring and the fork, so as to converge rays upon the slit between the plates, and a microscope placed before the fork to receive the rays when transmitted. This is focussed so that when the prongs of the fork are drawn apart a distinct image of one or more bands is seen. The fork itself is so placed that the slit when open is parallel to the length of the bands on the stroboscope.

The number of dark bands in each ring being known, and the stroboscope being rotated at a known velocity, the interval during which m teeth pass the cross-wire is noted by means of a stop-watch. That band should be chosen for observation, in which the motion of the teeth is sufficiently slow for counting.

If the noted interval of time be T, the vibrations of the fork meanwhile have been $pq\text{T} \pm m$.

The frequency of the fork consequently is given by—

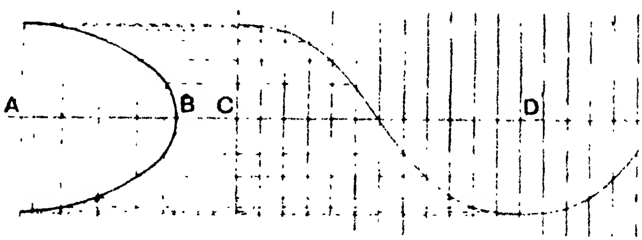
$$n = pq \pm \frac{m}{\text{T}}$$

the positive sign being taken if the movement of the teeth is against, the negative if they are with, the direction of rotation.

In practice, a stroboscope is more often used to determine an unknown velocity of rotation by a fork of known frequency.

184.

To determine the Frequency and Character of Vibration of a Vibrating String by Means of the Vibration Microscope.—The frequency may be determined by blackening the string and making a white spot upon it at the point to be observed, or by attaching a minute globule of mercury by grease, and adjusting it below the lens fork so that its vibrations are at right angles to those of the fork. The shape of the curve seen upon illuminating the spot and starting both string and fork varies, but the form repeats itself at intervals. The interval in



which the curve passes through one such cycle may be noted by a stop-watch, counting, of course, the total time of a number of cycles and dividing it by that number. In the period of one cycle the string has either gained or lost one vibration on the fork. Whether the former or the latter be the case may be ascertained by lowering the pitch of fork or string slightly until the noted period is increased The one of which the pitch has been lowered was originally that of greater frequency. The frequency of the lens fork being known, that of the string is thus determined.

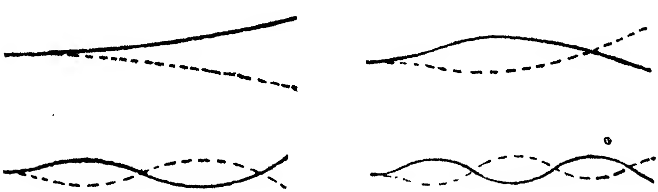
To determine the character of the string's vibration, fork and string are adjusted to the same frequency, either by

shifting the weights on the former or altering the length of the latter. The curve then seen is drawn as exactly as possible on a sheet of paper, and a line AB drawn midway between its extremities parallel to the direction of the fork's vibrations. A circle is described on AB as shown, and divided into twelve or more equal parts; the displacement of the string at corresponding intervals during one vibration is marked by the vertical distance between their projection on the curve and the axis AB. Let AB be produced and a length CD taken in it. If the same number of equal parts be taken on CD as in the circle, and parallel lines be drawn from the curve points, the displacement curve of the string may be constructed as shown, where abscissæ are times and ordinates displacements.

185.

To determine the Positions of the Nodes, and to compare the Frequencies of the Upper Partials of a Rod Clamped at One End and Vibrating Transversely.—The modes of vibration of a bar clamped in this manner are as shown, if the bar be of uniform density, elasticity, and section.

The most satisfactory method of determining the actual



positions of the nodes is to press a narrow edge firmly against the region of the rod in which one may be expected to occur, and to tap the rod where the antinode would be. A note is then emitted which becomes clear and sustained as the edge is adjusted to the position of the node. The clamping edge and the tapper should be of some soft material, such as wood, which does not of itself ring.

It is convenient to find firstly in each case that node which is nearest to the free end of the rod, afterwards finding the others, and noting the distances of each from the free end. In this way it is often possible to determine each set of nodes up to the fifth or sixth partial.

The clamped end in all cases must be regarded as a node, though neither this nor the node nearest the free end is a true node. The frequencies in any modes of vibration may be considered as inversely proportional to the square of the distance between the free end and its nearest node.

The position of the nodes may also be found by proceeding as above, and allowing a varnished pith-ball suspended by silk to touch the rod. The disturbance of the pith-ball is least at the nodes.

TABLES AND CONSTANTS

Tangents of Angles.

Degrees.	Tan.	Degrees.	Tan.	Degrees.	Tan.
0	0	17·0	0·30573	34·0	0·67451
0·5	0·00873	17·5	0·31530	34·5	0·68728
1·0	0·01748	18·0	0·32492	35·0	0·70021
1·5	0·02619	18·5	0·33460	35·5	0·71329
2·0	0·03492	19·0	0·34433	36·0	0·72654
2·5	0·04366	19·5	0·35412	36·5	0·73996
3·0	0·05241	20·0	0·36397	37·0	0·75355
3·5	0·06116	20·5	0·37389	37·5	0·76733
4·0	0·06993	21·0	0·38386	38·0	0·78129
4·5	0·07870	21·5	0·39391	38·5	0·79544
5·0	0·08749	22·0	0·40403	39·0	0·80978
5·5	0·09629	22·5	0·41421	39·5	0·82434
6·0	0·10510	23·0	0·42447	40·0	0·83910
6·5	0·11393	23·5	0·43481	40·5	0·85408
7·0	0·12278	24·0	0·44523	41·0	0·86929
7·5	0·13165	24·5	0·45573	41·5	0·88473
8·0	0·14054	25·0	0·46631	42·0	0·90040
8·5	0·14945	25·5	0·47698	42·5	0·91633
9·0	0·15838	26·0	0·48773	43·0	0·93252
9·5	0·16734	26·5	0·49858	43·5	0·94900
10·0	0·17633	27·0	0·50953	44·0	0·96569
10·5	0·18534	27·5	0·52057	44·5	0·98270
11·0	0·19438	28·0	0·53171	45·0	1·00000
11·5	0·20345	28·5	0·54296	45·5	1·0176
12·0	0·21256	29·0	0·55431	46·0	1·0355
12·5	0·22169	29·5	0·56577	46·5	1·0538
13·0	0·23087	30·0	0·57735	47·0	1·0723
13·5	0·24008	30·5	0·58905	47·5	1·0913
14·0	0·24933	31·0	0·60086	48·0	1·1106
14·5	0·25862	31·5	0·61280	48·5	1·1303
15·0	0·26795	32·0	0·62487	49·0	1·1504
15·5	0·27732	32·5	0·63707	49·5	1·1708
16·0	0·28675	33·0	0·64941	50·0	1·1918
16·5	0·29621	33·5	0·66189	50·5	1·2131

Degrees	Tan.	Degrees.	Tan.	Degrees.	Tan.
51·0	1·2349	64·5	2·0965	78·0	4·7046
51·5	1·2572	65·0	2·1445	78·5	4·9152
52·0	1·2799	65·5	2·1943	79·0	5·1446
52·5	1·3032	66·0	2·2460	79·5	5·3955
53·0	1·3270	66·5	2·2998	80·0	5·6713
53·5	1·3514	67·0	2·3559	80·5	5·9758
54·0	1·3764	67·5	2·4142	81·0	6·3138
54·5	1·4019	68·0	2·4751	81·5	6·6912
55·0	1·4281	68·5	2·5386	82·0	7·1154
55·5	1·4550	69·0	2·6051	82·5	7·5958
56·0	1·4826	69·5	2·6746	83·0	8·1443
56·5	1·5108	70·0	2·7475	83·5	8·7769
57·0	1·5399	70·5	2·8239	84·0	9·5144
57·5	1·5697	71·0	2·9042	84·5	10·385
58·0	1·6003	71·5	2·9987	85·0	11·430
58·5	1·6319	72·0	3·0776	85·5	12·706
59·0	1·6643	72·5	3·1716	86·0	14·301
59·5	1·6977	73·0	3·2709	86·5	16·350
60·0	1·7321	73·5	3·3759	87·0	19·081
60·5	1·7675	74·0	3·4874	87·5	22·9040
61·0	1·8040	74·5	3·6059	88·0	28·636
61·5	1·8418	75·0	3·7321	88·5	38·188
62·0	1·8807	75·5	3·8667	89·0	57·290
62·5	1·9210	76·0	4·0108	89·5	114·59
63·0	1·9626	76·5	4·1653	90·0	∞
63·5	2·0057	77·0	4·3315		
64·0	2·0503	77·5	4·5107		

Comparisons of Units.

1 centimetre = 0·3937 inch.	1 inch = 2·54 centimetres.
1 metre = 39·37079 inches.	1 foot = 30·48 ,,
1 square centimetre = 0·155 square inch.	1 square inch = 6·451 square centimetres.
1 cubic centimetre = 0·061 cubic inch.	1 cubic inch = 16·38 cubic centimetres.
1 kilogram = 2·205 pounds.	1 pound = 453·59 grams.

Density of Water at Various Temperatures.

Temp. C.°	Density.	Temp. C.°	Density.
0	0·99988	17	0·99885
1	0·99994	18	0·99867
2	0·99998	19	0·99847
3	0·99999	20	0·99827
4	1·00000	21	0·99806
5	0·99999	22	0·99784
6	0·99998	23	0·99761
7	0·99995	24	0·99738
8	0·99990	25	0·99713
9	0·99984	30	0·99578
10	0·99976	40	0·99236
11	0·99967	50	0·98821
12	0·99956	60	0·98339
13	0·99944	70	0·97795
14	0·99931	80	0·97195
15	0·99917	90	0·96557
16	0·99901	100	0·95866

Density of Mercury at Various Temperatures.

Temp. C.°	Density.	Temp. C.°	Density.
0	13·596	50	13·474
5	13·584	60	13·450
10	13·572	70	13·425
15	13·560	80	13·400
20	13·547	90	13·377
30	13·525	100	13·353
40	13·500		

Densities.

Lead	11·4	Glass, crown ...	2·5–2·7
Silver	10·5	Aluminium ...	2·60
Copper ...	8·5–8·9	Beeswax ...	0·96
Brass	8·1–8·6	Cork ...	0·24
Iron, wrought	7·8	Turpentine	0·87
Iron, cast ...	7·1–7·6	Alcohol	0·80
Glass, flint ...	3·0–5·0	Glycerine	1·26

Densities of Gases at 0° and 760 mm.

1 litre of hydrogen weighs 0·0896 gram.
1 litre of dry air weighs 1·2932 grams.

Young's Modulus.

Steel wire ...	...	20×10^8 grams.
Drawn copper	...	12×10^8 ,,
Brass, cast ...	...	$6·4 \times 10^8$,,
Brass wire ...	...	10×10^8 ,,
Oak	...	1×10^8 ,,
Mahogany ...	...	$0·9 \times 10^8$,,

Surface Tensions, in Dynes, at 20° C.

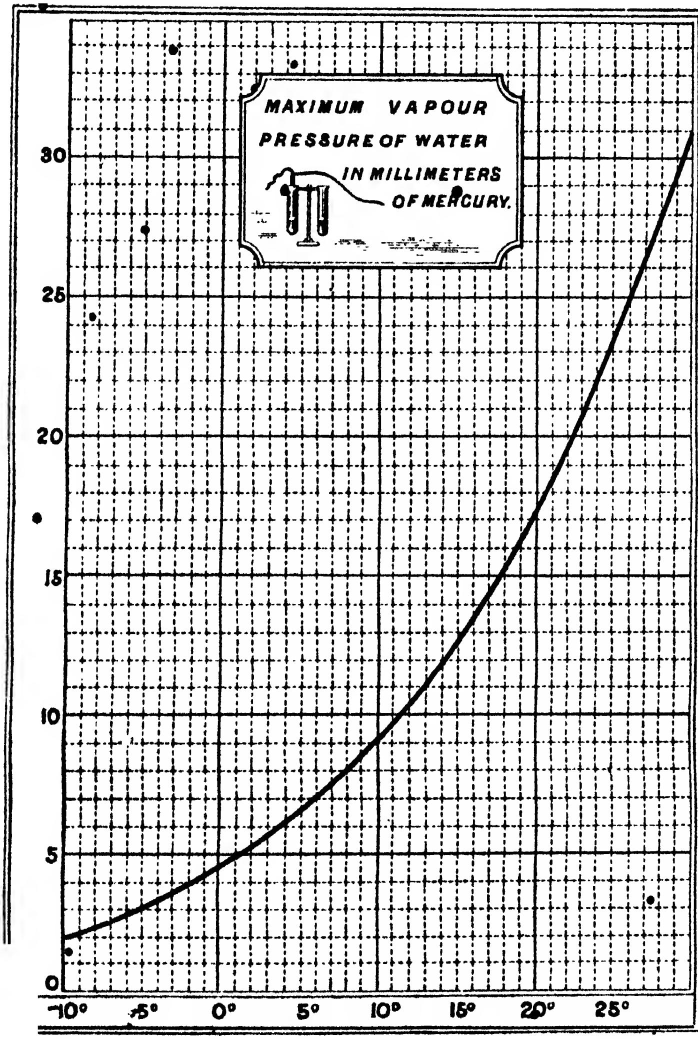
Water	73
Alcohol	26
Turpentine	29
Mercury	450
Mercury and water	350
Turpentine and water	11
Olive oil and water	20

Viscosities, in Dynes.

Water at 10° C. ...	0·013	Olive oil at 10° C.	0·4200
,, ,, 20° C. ...	0·010	Linseed oil at ,,	0·1540
Mercury at 10° C. ...	0·016	Alcohol at ,,	0·0016

Water-vapour Pressure.

Temp. C.°	Millimetres of *Hg*.
0	4·6
5	6·53
10	9·17
15	12·7
20	17·4
25	23·55
30	31·55



COEFFICIENTS OF LINEAR EXPANSION.

Brass, 0·000019; glass, about 0·000009; steel, about 0·000011.

Specific Heats.

Copper	= 0·095	Mercury	= 0·034
Brass	= 0·094	Alcohol	= 0·580
Glass	= 0·200	Aniline	= 0·490
Iron	= 0·112	Turpentine	= 0·430
Lead	= 0·030	Air, const. pres.	= 0·238

Magnetic Elements.

H at London = 0·18 | H at Bradford = 0·172

Specific Resistances.

Copper ...	$1·64 \times 10^{-6}$ ohms.	Iron ...	$9·8 \times 10^{-6}$ ohms.
Aluminium	$2·9 \times 10^{-6}$,,	Mercury	$94·1 \times 10^{-6}$,,
Platinum	$9·16 \times 10^{-6}$,,		

Specific Inductive Capacities.

Ebonite = 2·2–4·0	Glass = 6·6–9·8
Paraffin = 1·95–2·0	Mica = 6·0–6·6

Electro-chemical Equivalents.

1 coulomb of electricity liberates 0·00033 gram of copper.
1 ,, ,, 0·0000105 gram of hydrogen.

Wave-lengths of Light.

Sodium	5895 and 5889 $\times 10^{-8}$	centimetres.
Lithium	6705 ,, 6102 $\times 10^{-8}$	,,
Potassium	7680 ,, 4043 $\times 10^{-8}$	,,
Strontium { longest red ...	6627 $\times 10^{-8}$	,,
Strontium { blue	4607 $\times 10^{-8}$	,,

Refractive Indices.

Crown glass ...	1·5	Glycerine	1·47
Flint glass	1·6	Carbon bisulphide ...	1·61
Water	1·33	Turpentine	1·48

Rotation of Polarized Sodium Light. N per cent. Solutions, at 20° C.

Cane sugar	+ ·0665 N° per cm. thickness.
Milk sugar	+ ·0525 N° ,, ,,

Velocity of Sound and Frequency.

Velocity = 332 + 0·6 (temp. C.°) metres per second in air.
A = 440, C = 528, complete vibrations per second.

APPENDIX

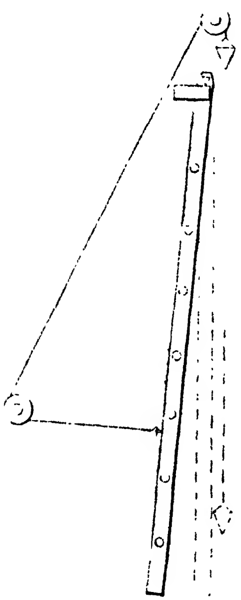
16A

To determine the Density of a Liquid by Mohr's Balance. —This balance, which affords an extremely accurate means of finding the density of liquids, consists essentially of a lever, or beam, weighted at one end. This movable beam carries at one end a hook, from which is suspended, by a fine platinum wire, a glass displacement vessel, and on which weights may be suspended. At the other end is a metal point, which in the position of equilibrium is directly level with another fixed point. The arm by which the weights are carried is divided into ten parts, so that a weight, when placed on any division—suppose the mth—is reckoned as m tenths of its value when on the hook. The weights themselves are commonly such that their ratios are 3, 2, 1, 1, 0·1, 0·01, and, if the balance be in order, the third of these, suspended from the hook, should produce equilibrium with the sinker in pure water at 15° C. The two first weights are only used in the case of liquids denser than water. In use, the stand should originally be levelled by the aid of the plumb-line provided, and the subsequent manipulation of the weights be methodically performed as in an ordinary weighing. It will be evident that, with the weights mentioned, the range will be from 0·001 by thousandths up to 7·11, though the sensibility will depend very

largely on the surface tension and viscosity of the liquid under consideration.

30A.

To determine the Acceleration of Gravity by Means of a Pendulum and Falling Body.—The simple apparatus required for this determination consists of a wooden rod about 4 feet long, furnished at one end with knife-edges. At intervals along its length holes should be bored, into which leaden cylinders may be inserted, so as to vary the period of vibration.



The falling body should be of metal and in the form of a double cone, while the bearings for the knife-edges and the small pulleys shown in the diagram may be fitted on a framework or conveniently attached to a wall. The pendulum should firstly be adjusted, so that, when hanging vertically, the front face is just in contact with the edge of the weight suspended from its pulley. A fine thread is then attached to the weight and pendulum, its length being such that, when placed over the two pulleys, the weight is near the top of the pendulum, which itself is drawn aside to a small extent. A narrow strip of paper should now be attached to the front face of the pendulum, and the sharp median edge of the weight should be rubbed with ink or black grease. The vertical height from any fixed point to the edge of the weight is then measured: let this be H. Upon burning the thread above the lower pulley, the weight falls, and its edge makes a mark upon the paper with

which it comes in contact as soon as the pendulum reaches a vertical position. Let the vertical height measured from the previous fixed point to the top of this mark be h. Then the weight has fallen a distance $H-h$, while the pendulum has made one-fourth of a vibration.

The period of vibration is now found by the method of Art. 27. If this be T, the space passed over by a falling body in the time $\frac{T}{4}$ is $(H-h)$. The same operations should be repeated with the leaden cylinder placed in the holes, one after another, downwards, so as to increase the time of vibration, and results tabulated thus—

	h.	$H-h$.	$\frac{1}{4}T$.	$\left(\frac{T}{4}\right)^2$

Finally, a curve should be set out with values of $\left(\frac{T}{4}\right)^2$ as abscissæ, and of $(H-h)$ as ordinates. Taking, then, any point on this curve, and noting the values of $\left(\frac{T}{4}\right)^2$ and $(H-h)$ there, g is obtained. For—

$$\text{Space fallen} = \tfrac{1}{2}g(\text{time in secs.})^2$$

$$H - h = \tfrac{1}{2}g\left(\frac{T}{4}\right)^2$$

$$g = \frac{2(H-h)}{\left(\frac{T}{4}\right)^2}$$

91A.

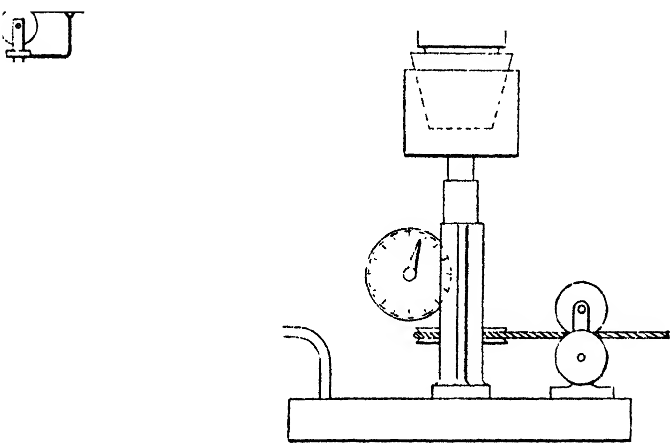
To determine the Mechanical Equivalent of Heat.—The mechanical equivalent of heat, known also as Joule's equivalent, is the number of ergs of mechanical work which must be expended to raise the temperature of 1 gram of water by 1° Centigrade. The apparatus shown in the diagram is a modification of that originally adopted by Pulaj, wherein work is expended in rotating a brass conical cup against the friction of a well-fitting inner one which is stationary. The latter contains a known weight of water; and its noted rise of temperature, together with a knowledge of its water equivalent, enables the number of calories generated to be ascertained. The mechanical energy expended is learned by noting the number of revolutions made by the cup against a constant torque.

The two brass cups are firstly weighed, and from a knowledge of the specific heat of brass (0·094), their water equivalent is found. Let this, together with the estimated equivalent of the thermometer bulb, be WEq. The inner cup is then filled with a measured volume of water to within about 1·5 cm. from the top, and both cups are replaced on the driving spindle, two or three drops of oil being wiped over their rubbing surfaces. The wooden disc and weight are added, and a sensitive thermometer supported by a clip, with its bulb in the water. To a peg on the circumference of the disc is attached a fine cord, which passes round the disc, leaving it tangentially to run over a small pulley to a weight, M (of about 250 grams), and a second weight, N.

The driving cords, arranged on the guide pulleys as shown, pass to a driving wheel turned by hand, and the number of revolutions made by the cup is indicated by a toothed wheel and index, the former making one revolution per hundred of the cup. These arrangements and measurements being made, the temperature of the water is read by one observer, who also keeps the water stirred and notes the revolutions made, while

the driving wheel is turned by another person. The turning of this requires considerable care, as the speed must be so adjusted that M is suspended by its cord while N is resting on the floor. To facilitate this it is well to attach a large paper pointer to the wooden disc, and to see that it only oscillates to and fro as little as possible about some fixed position. As soon as the temperature has risen by about 20°, the operations may cease.

Where possible, it is most satisfactory to fill the cup with water at about 10° lower temperature than that of the room,



letting it finally attain a temperature about 10° higher. If, however, it be, on starting, at the same temperature as the room, a correction for the loss of heat by radiation, etc., should be applied. To this end the rate at which the water in the cup cools should be noted as soon as the previous operations stop, and the total time of rotation should also be observed. If θ_2 be the highest temperature shown by the thermometer, and if it be noted to fall at the rate $\theta°$ per minute on stopping, then, if t be the number of minutes

over which the rotation extended, the correction is made by taking

$$\theta_2 + \tfrac{1}{2}\theta t$$

instead of θ_2, as the final temperature.

Indicating now the equivalent by J, the weight in grams of water in the cup by W, the radius of the wooden disc by r, and the number of rotations of the cup by n, WEq being the water equivalent of the cups, and M the weight in grams of the suspended mass, θ_1 also denoting the original temperature of the water (equal to that of the air), the ergs expended are equal to—

$$2\pi n r \mathrm{M} \times 981$$

and the calories produced equal—

$$(\theta_2 + \tfrac{1}{2}\theta t - \theta_1)(\mathrm{W} + \mathrm{WEq})$$

Hence, J being the ratio $\dfrac{\text{ergs}}{\text{calories}}$

$$\mathrm{J} = \frac{2 \times 981 \times \pi n r \mathrm{M}}{(\theta_2 + \frac{1}{2}\theta t - \theta_1)(\mathrm{W} + \mathrm{WEq})}$$

or, in the case of originally cooled water, where no correction to θ_2 is necessary—

$$\mathrm{J} = \frac{2 \times 981 \times \pi n r \mathrm{M}}{(\theta_2 - \theta_1)(\mathrm{W} + \mathrm{WEq})}$$

θ_1 in this expression indicating the original temperature of the water, which is lower than that of the room.

155.

(*d*) The focal length of a thin convex lens may be ascertained by laying it upon a plane mirror and supporting a needle or pointed piece of paper vertically above it. The latter is adjusted in height until, when viewed from above, the point and its image coincide, and do not separate as the eye is

moved to and fro, *i.e.* until there is no parallax. In this condition the distance from the point to the centre of the lens's thickness is the focal length of the lens, for rays from the object after traversing the lens are parallel, and are reflected as parallel rays, giving an inverted image in the horizontal plane of the object. This method is particularly suited for lenses of considerable focal length.

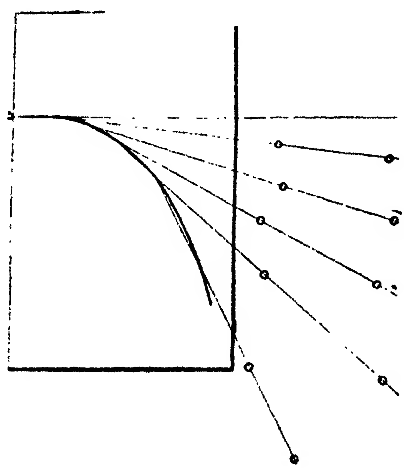
156.

(*c*) The focal length of a concave lens may be determined also by so placing it in a stand that the image of some distant object is seen through it. The position of this virtual image is, of course, at the principal focus. To find the distance of the latter from the lens, let a brightly illuminated object or a gas-jet be placed between the lens and the eye, and a plate of plane glass be erected in a vertical plane between the luminous object and the lens. The eye being then placed on the axis of the lens, the position of the glass is altered until the image of the luminous object formed by it and the image of the distant object formed by the lens are seen without parallax. Then the principal focus of the lens is as far behind the sheet of glass as the illuminated object is in front of it.

157A.

To plot the Caustic Curve of Rays obliquely reflected from a Spherical Surface, or refracted through a Plane Surface.—The caustic curve of reflected rays may be seen as a bright line if a thin strip of bright metal be bent to the radius of the spherical surface considered, and laid upon a sheet of white paper, with a small source of light (*e.g.* a miniature glow-lamp) placed at some point on the axis of the curve. This curve may be marked on the paper for various positions of the lamp, and should, in particular, be so marked when the lamp is at the centre of curvature and midway between this point and the reflecting surface.

The caustic of refracted rays may be obtained by placing a pin vertically in contact with the back of a thick block of glass and viewing it at various angles of obliquity through the front face. At each position two other pins are placed in line, with the image of the pin at the back, as in Art. 153, and, after removing the glass block, lines are drawn through each



pair of pins. These lines intersect each other, and the curve drawn through all the points of intersection is the caustic curve.

160.

(*b*) The focal length and principal points of a thick convex lens may also be found in the following manner:—The lens, mounted in a suitable stand, is placed between illuminated cross-wires and a screen, its position between them being adjusted until a real image of the wires is given on the screen. In general, there are two possible positions, giving images less and larger respectively than the object. If, however, the wires and screen be brought nearer together, these positions approach

each other, and ultimately coincide, there being then a single position for the lens, and the object and image being of equal size.

Under these circumstances, the distance between the wires and screen is equal to four times the focal length, plus the distance between the principal points. The screen is at a distance equal to twice the focal length from the second principal point, and its distance from the second principal focus is equal to the focal length.

Now, the position of the second principal focus relatively to the lens face may be found by the method of Art. 155 (*d*). Let its distance from the upper face, which is to be toward the screen, be l, and the measured distance between the nearer face of the lens and the screen be D, then the focal length F, reckoned from the principal point, is obtained from the equation—

$$F = D - l$$

and the position of the second principal point may be marked off on the edge of the lens, or on its mounting, at a distance $2(D - l)$ from the screen.

By finding the principal focus for the face of the lens toward the cross-wires, and making similar measurements from the latter instead of from the screen, the position of the other principal point may also be determined.

(*c*) The fact that, as pointed out on p. 266, an incident ray directed towards one principal point of a lens emerges in a parallel direction from the other, is the basis of a method, devised by Clay, for the determination of the nodal or principal points.

The lens is carried, with its axis horizontal, on a carriage or table capable of revolving about a vertical axis. Before the lens is placed a vertical screen, in which is an aperture with illuminated cross-wires, and immediately behind the lens a plane mirror is fixed vertically. The screen is firstly adjusted at such a distance from the lens that a real image of the cross-wires is formed in the neighbourhood of the aperture; and this

condition being attained, the cross-wires are at the first focal point (Art. 155 (*d*)). The rotation of the lens table about the vertical axis will now, in general, produce a displacement of the image; but by moving the lens forward or backward on its table, so that the distance between its front face and the vertical axis is varied, a position is found at which rotation of the table produces no displacement of the image. The screen must also be adjusted each time the lens is moved, so that its distance from the latter is kept constant. Under these conditions, the first principal point of the lens and the axis of rotation lie in the same vertical plane, and its position may be marked. Reversing the faces of the lens, the other principal point may be similarly found.

(*d*) In the case of a concave, or divergent, lens, the method of 160 (*c*) may be applied, the only difference being that a convex lens of sufficiently short focal length must be placed in some fixed position between the rotating lens table and the screen. The focal length of the concave lens is found by removing it from the rotating table, laying aside the plane mirror, and noting by a second screen the position of the real image of the cross-wires given by the convex lens. The distance between this image and the table's axis of rotation is then the focal length.

45 (*Note*).

Since the meniscus is brought to its original position on raising the beaker, the distance from it to the level in the beaker is the height of the column supported by the capillarity of the tube. Hence the height, H, through which the beaker was raised, was the column supported by the drop. The downward pressure on the latter was then approximately—

$$\tfrac{1}{4}\pi\rho D^2 H$$

This force, tending to tear the lower half of the drop away,

is balanced by the total tension πDT round the circumference. Thus—

$$\pi DT = \tfrac{1}{4}\pi\rho D^2 H$$
$$T = \frac{\rho DH}{4}$$

ρ being the density of the liquid.

46 (*Note*).

The expression for T, as given on p. 73, may be obtained as follows:—

If R, R′ be at any point the principal radii in perpendicular directions of a curved interface between two media of densities ρ_1 and ρ_2; and h the height of the point, measured from the lowest portion of the surface—

$$T\left(\frac{1}{R} + \frac{1}{R'}\right) = h(\rho_1 - \rho_2) \quad . \quad . \quad . \quad . \quad \text{(i)}$$

and if one radius, R′, be so large that its reciprocal is negligible

$$h = \frac{T\left(\frac{1}{R}\right)}{\rho_1 - \rho_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}$$

Now, if x be measured horizontally, the curvature at the point is given by

$$\frac{1}{R} = \frac{\frac{d^2h}{dx^2}}{\left(1 + \left(\frac{dh}{dx}\right)^2\right)^{\frac{3}{2}}} \qquad \text{(iii)}$$

as shown in most works on the calculus (Perry's "Calculus," p. 307). Hence, if the density of one medium be neglected, and that of the other regarded as ρ, from (ii.) and (iii.)—

$$\rho h = \frac{Th''}{(1 + h'^2)^{\frac{3}{2}}}$$

and—

$$\rho h - \frac{Th''}{(1 + h'^2)^{\frac{3}{2}}} = 0$$

Multiplying by h' and integrating—

$$\frac{\rho h^2}{2} + \frac{T}{(1 + h'^2)^{\frac{1}{2}}} = C \quad . \qquad \text{(iv)}$$

If θ be the angle which the tangent drawn to the surface at the point makes with the vertical—

$$\sin \theta = \frac{\quad}{(1 + h'^2)^{\frac{1}{2}}}$$

so that (iv.) may be written—

$$T \sin \theta + \frac{\rho h^2}{2} = C$$

To determine the constant C, when $h = 0$, the value of θ is $90°$; therefore $C = T$. Thus—

$$T = \frac{\rho h^2}{2(1 - \sin \theta)}$$

Now, if a vertical tangent be drawn to touch the curve of a drop (Art. 46, Fig.), $\sin \theta = 0$, and the height of column producing the pressure at the point is $h_1 - h_2$, or H. Whence—

$$T = \frac{H^2 \times \text{density}}{\quad}$$

48 (*Note*).

The expression for the viscosity as given on p. 74 may be deduced as follows:—Since the flow is uniform, the force on any cylindrical surface of radius r and unit length, coaxial with

the tube, is $2\pi r\mathrm{M}\frac{dv}{dr}$, v denoting the velocity of flow. The rate of variation of this with respect to r is $2\pi\mathrm{M}\frac{d}{dr}\left(r\frac{dv}{dr}\right)$; or, considering an annular element of radial width δr, since there is no acceleration—

$$\text{pressure} = 2\pi\mathrm{M}\frac{d}{dr}\left(r\frac{dv}{dr}\right)\delta r$$

and the pressure per unit axial length is $-\left(2\pi r\delta r\frac{dp}{dx}\right)$.

Since the motion is slow, p is independent of r, and $\frac{dp}{dx} = \frac{\mathrm{P}}{\mathrm{L}}$. Hence—

$$r\frac{\mathrm{P}}{\mathrm{L}} = -\mathrm{M}\frac{d}{dr}\left(r\frac{dv}{dr}\right)$$

and, integrating—

$$-\frac{dv}{dr} = \frac{\mathrm{P}r}{2\mathrm{LM}}$$

$$-v = \left[\frac{\mathrm{P}r^2}{4\mathrm{LM}}\right]_{r_1}^{r_2}$$

for the entire annular element.

The volume flowing through the annular element in unit time is $2\pi rvdr$. Hence integrating between $r = 0$ and $r = \mathrm{R}$—

$$\pi\int_0^{\mathrm{R}} rvdr = \frac{\pi\mathrm{P}}{2\mathrm{ML}}\int_0^{\mathrm{R}} r(r^2)dr = \frac{\pi\mathrm{P}}{8\mathrm{M}}$$

THE END

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